Whitevox

4 PRACTICE PAPERS 2022

JEE Main | Advanced

www.mtg.in | May 2022 | Pages 80 | ₹ 40

Class XII **CBSE** warm up! TERM-II

MATHEMATICS today

India's #1
MATHEMATICS MONTHLY
for JEE (Main & Advanced)

OLYMPIAD CORNER

PRACTICE PAPER 2022
CUET (UG)

JEE WORK OUTS

CONCEPT MAP
CLASS XI-XII

MONTHLY TEST DRIVE
CLASS XI-XII

CELEBRATING 40 YEARS OF EXCELLENCE

mtG
Trust of more than
1 Crore Readers
Since 1982

2022500013383

BEAT THE TIME TRAP

You Ask ?
We Answer ✓

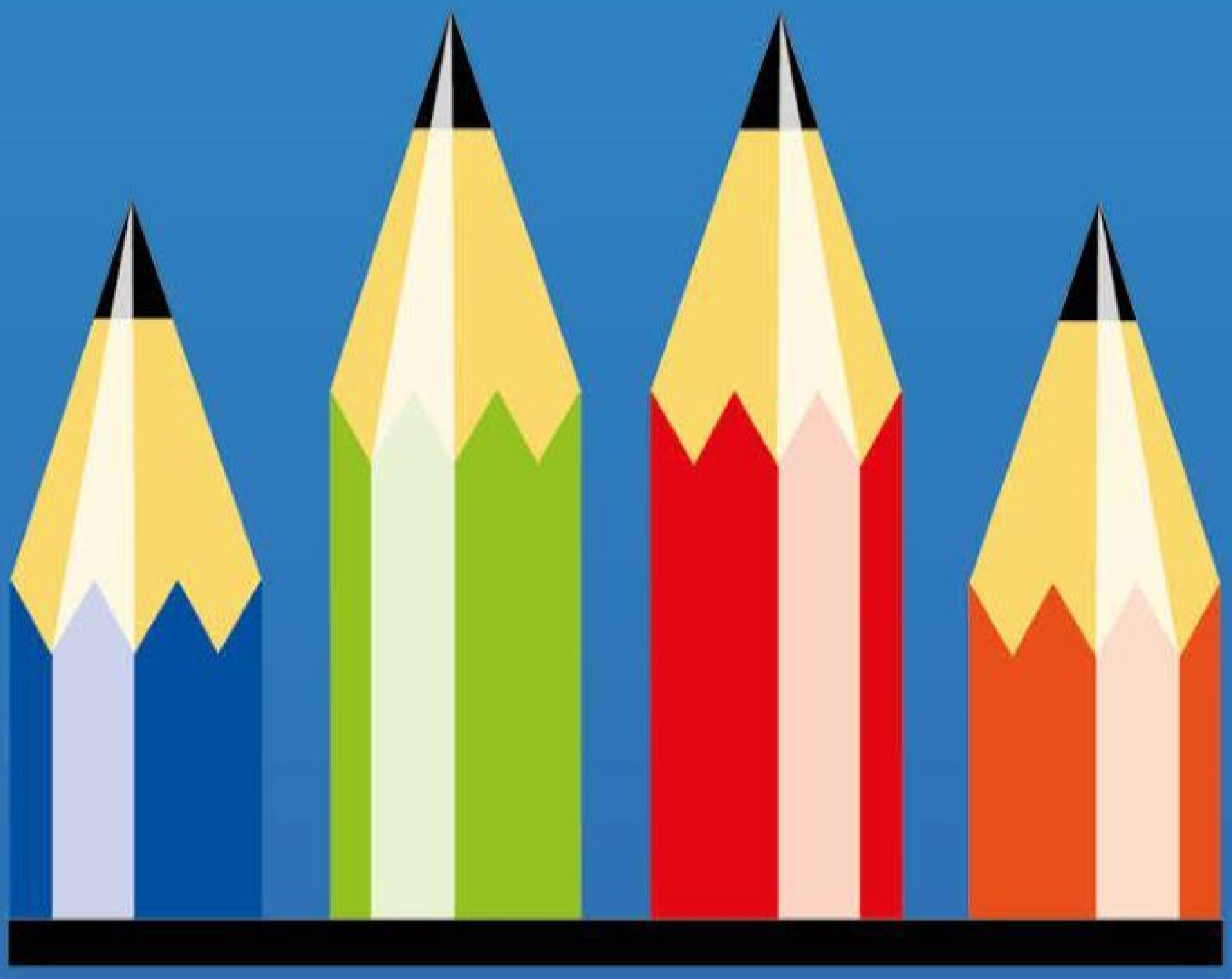
AWARDS* & PRIZES

- Total awards / scholarships during 2019-20 approx. ₹18 Crores
- 30% increase in Zonal Awards
- Gold Medals of Excellence to top 10% in class
- Certificates & Performance Analysis Reports for all schools & students

SOF
SCIENCE OLYMPIAD FOUNDATION
INSPIRING YOUNG MINDS THROUGH KNOWLEDGE OLYMPIADS

ANNOUNCES

Important: Applications must reach us by **August 31, 2022.**



THE WORLD'S BIGGEST OLYMPIADS

2022-23

Millions of students from 68,000+ schools in 1400+ cities and 48 countries competing in the 7 SOF Olympiads!

FOR DETAILS

- Visit www.sofworld.org
- Visit www.facebook.com/sofworld
- Write to us at info@sofworld.org
- Call : 0124 - 4951200
- Check with your teacher

SCHOLARSHIPS

- Girl Child Scholarship Scheme - 300 Nos. ₹ 5,000 each
- Defence Services *New* Academician Scholarship-260 Nos. ₹ 5,000 each +Trophy
- Academic Excellence Scholarship-260 Nos. ₹ 5,000 each +Trophy

THE INSTITUTE OF
Company Secretaries of India
भारतीय कम्पनी सचिव संस्थान
IN PURSUIT OF PROFESSIONAL EXCELLENCE
Statutory body under an Act of Parliament
PRESENTS

IGKO	SOF INTERNATIONAL GENERAL KNOWLEDGE OLYMPIAD	15 SEPT. | 11 OCT.
IEO	SOF INTERNATIONAL ENGLISH OLYMPIAD	22 SEPT. | 14 OCT. | 1 NOV.
NSO	SOF NATIONAL SCIENCE OLYMPIAD	18 OCT. | 3 NOV. | 1 DEC.
IMO	SOF INTERNATIONAL MATHEMATICS OLYMPIAD	10 NOV. | 22 NOV. | 13 DEC.
NCO	SOF NATIONAL CYBER OLYMPIAD	24 NOV. | 20 DEC.
ISSO	SOF INTERNATIONAL SOCIAL STUDIES OLYMPIAD	22 DEC. | 24 JAN.
ICO	SOF INTERNATIONAL COMMERCE OLYMPIAD	22 DEC. | 24 JAN.

✓Plot No. 99, First Floor, Sector 44, Gurugram -122 003 (HR) Telefax: (0124)4951200, 09312680857/55

Regd. Office: 406, Taj Apt., Ring Road, New Delhi-110 029 | e-mail: info@sofworld.org

**T&C apply, please refer to prospectus & our site for details.*

Disclaimer : *The content available in MTG digital/physical books is intellectual property of MTG Learning Media Pvt Ltd. Paid content on this website after purchase is not entitled for free distribution on whatsapp, telegram, twitter, facebook, and all other social media and digital channels. The registered owner/admin/ individual of any digital channel will be liable for an infringement of the copyright for illegally using the material / copyrighted content of MTG Learning Media Pvt Ltd.*

CBSE BOARD 2021-22
TERM-II

Based on Latest
Competency Based
Education (CBE)
12

MtG
ScoreMore
21 SAMPLE PAPERS
✓ Sample Question Papers (SQPs) with BLUEPRINT as design issued by CBSE
✓ Latest CBSE Term-II (2021-22) Sample Question Paper
✓ Self Evaluation Sheet included to check your readiness

Price
₹ 200/-
each

STRICTLY BASED ON
LATEST PATTERN AND
REVISED SYLLABUS

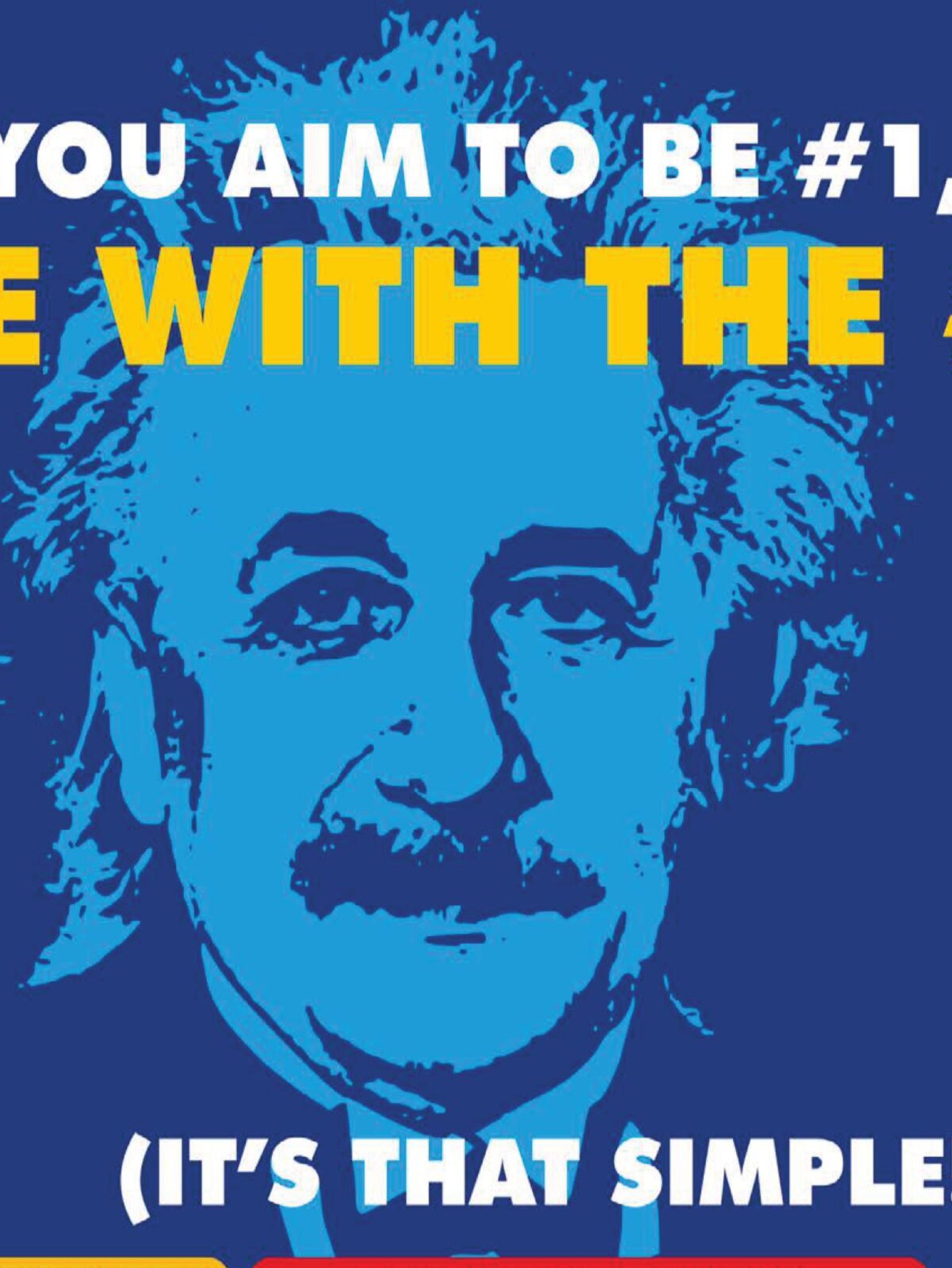
12
CBSE BOARD 2021-22
TERM-II
ScoreMore
21 SAMPLE PAPERS
PHYSICS
CHEMISTRY
MATHEMATICS
BIOLOGY
ENGLISH CORE

Available at all leading book shops throughout India.
For more information or for help in placing your order,
Call 0124-6601200 or e-mail:info@mtg.in

Strictly Based on the design of Sample Question Paper
issued by CBSE in January 2022

mtG
Success Made Simple

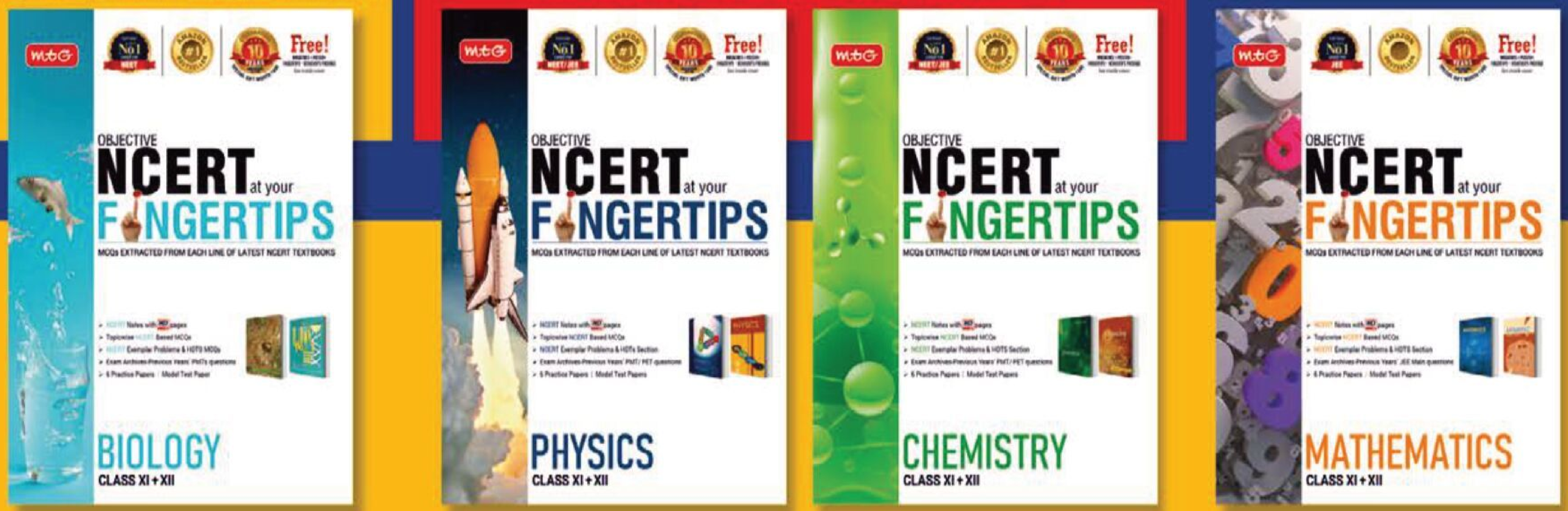
IF YOU AIM TO BE #1,
BE WITH THE #1.



(IT'S THAT SIMPLE!)

#1 BESTSELLER ON AMAZON

AMONG TOP 10 AMAZON BESTSELLERS


OBJECTIVE NCERT at your FINGERTIPS
MCQs EXTRACTED FROM EACH LINE OF LATEST NCERT TEXTBOOKS
BIOLOGY CLASS XI + XII
PHYSICS CLASS XI + XII
CHEMISTRY CLASS XI + XII
MATHEMATICS CLASS XI + XII
Free!

WHAT MAKES US #1:

- Chapterwise and topicwise student-friendly NCERT Notes with High Definition (HD) Multicolour pages
- Huge variety of questions framed from each and every line of NCERT textbooks • NCERT figure based questions included
- MCQs arranged topicwise and aligned with sequence of NCERT content to check your progress
- NCERT Exemplar MCQs • Assertion & Reason questions for giving an edge to your preparation
- Thinking Corner which includes HOTS MCQs and Numerical Value Type Questions
- Exam Archive includes Previous Years' NEET, PMTs and JEE Main questions
- Six Practice papers and Model Test Papers of NEET and JEE for self assessment

Call 0124-6601200/1800-10-38 673 or email info@mtg.in to learn more

MATHEMATICS today

Vol. XL No. 5 May 2022

Corporate Office:
Plot 99, Sector 44 Institutional Area,
Gurugram -122 003 (HR), Tel : 0124-6601200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.
Managing Editor : Mahabir Singh
Editor : Anil Ahlawat

CONTENTS

Competition Edge

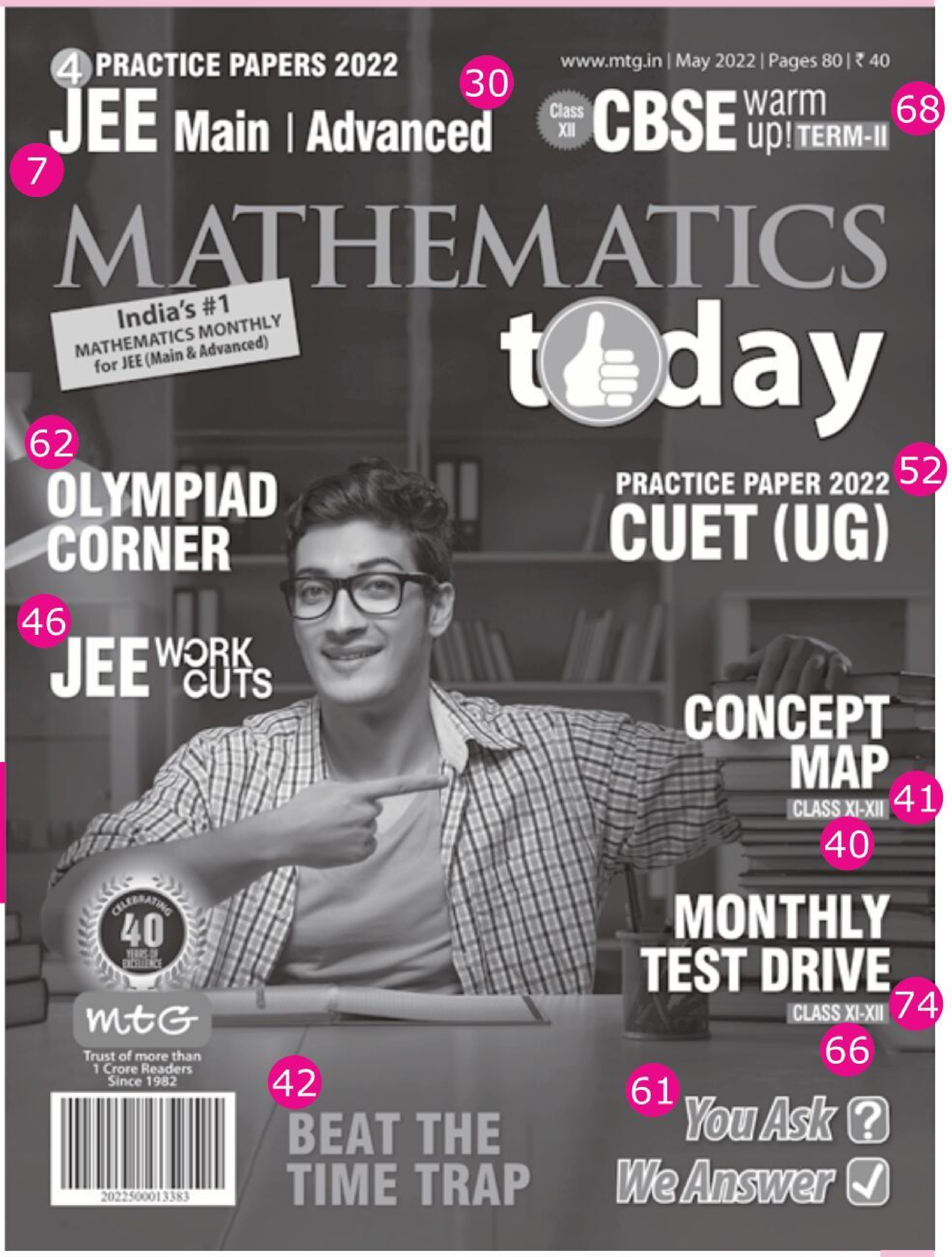
7 JEE Main
Practice Paper 2022
13 Gear Up for JEE Main
Practice Paper 2022
23 JEE Main
Mock Test Paper 2022
30 JEE Advanced
Practice Paper 2022
42 Beat The Time Trap
46 JEE Work Outs
48 Challenging Problems
52 CUET (UG)
Practice Paper 2022
61 You Ask We Answer
62 Olympiad Corner

Class XI

40 Concept Map
Statistics
66 Monthly Test Drive
Practice Paper

Class XII

41 Concept Map
Indefinite Integrals
68 CBSE Warm Up!
Practice Paper 2022
74 Monthly Test Drive
Practice Paper


4 PRACTICE PAPERS 2022
JEE Main | Advanced
7
30
www.mtg.in | May 2022 | Pages 80 | ₹ 40
Class XII CBSE warm up! TERM-II
68
MATHEMATICS today
India's #1 MATHEMATICS MONTHLY for JEE (Main & Advanced)
62
OLYMPIAD CORNER
PRACTICE PAPER 2022 CUET (UG)
52
46
JEE WORK OUTS
CONCEPT MAP CLASS XI-XII
41
40
MONTHLY TEST DRIVE CLASS XI-XII
74
66
CELEBRATING 40 YEARS OF EXCELLENCE
MtG Trust of more than 1 Crore Readers Since 1982
42
BEAT THE TIME TRAP
61
You Ask ? We Answer ✓

Individual Subscription Rates

	9 months	15 months	27 months
Mathematics Today	300	500	850
Chemistry Today	300	500	850
Physics For You	300	500	850
Biology Today	300	500	850

Combined Subscription Rates

	9 months	15 months	27 months
PCM	900	1400	2500
PCB	900	1400	2500
PCMB	1200	1900	3400

Send D.D/M.O in favour of MTG Learning Media (P) Ltd.
Payments should be made directly to : MTG Learning Media (P) Ltd,
Plot 99, Sector 44 Institutional Area, Gurugram - 122 003, Haryana.
We have not appointed any subscription agent.

Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
Editor : Anil Ahlawat

Readers are adviced to make appropriate thorough enquiries before acting upon any advertisements published in this magazine. Focus/Infocus features are marketing incentives. MTG does not vouch or subscribe to the claims and representations made by advertisers. All disputes are subject to Delhi jurisdiction only.
Copyright© MTG Learning Media (P) Ltd.
All rights reserved. Reproduction in any form is prohibited.

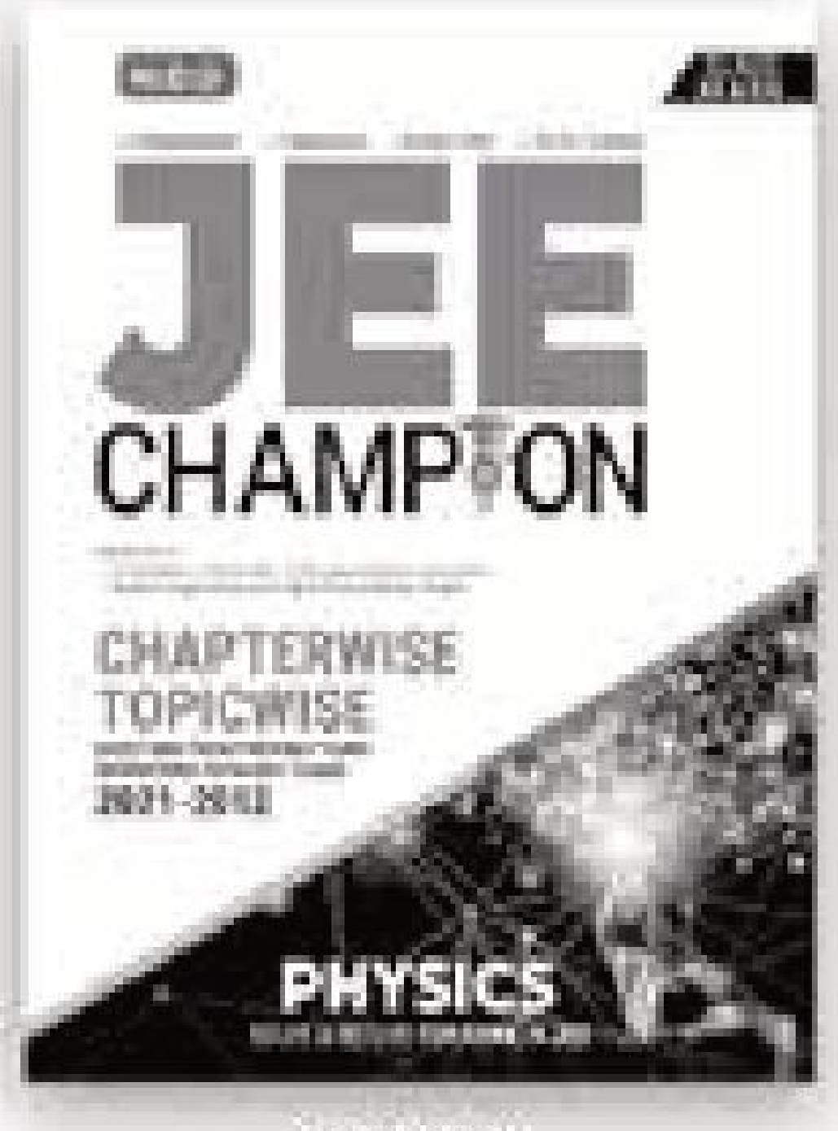
mtG

For Enquiries
0124-6601200

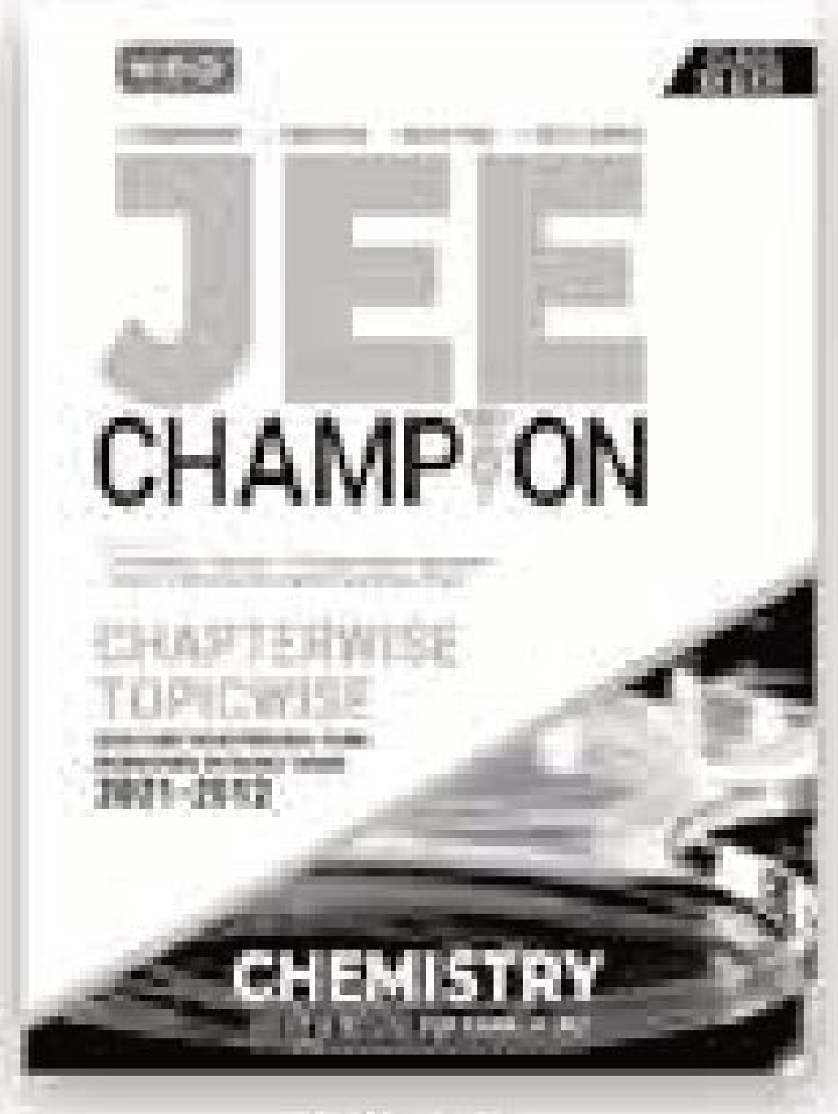
Find all your QUESTIONS here, before getting them in your exam.

By not studying
MORE but
ACCURATE...

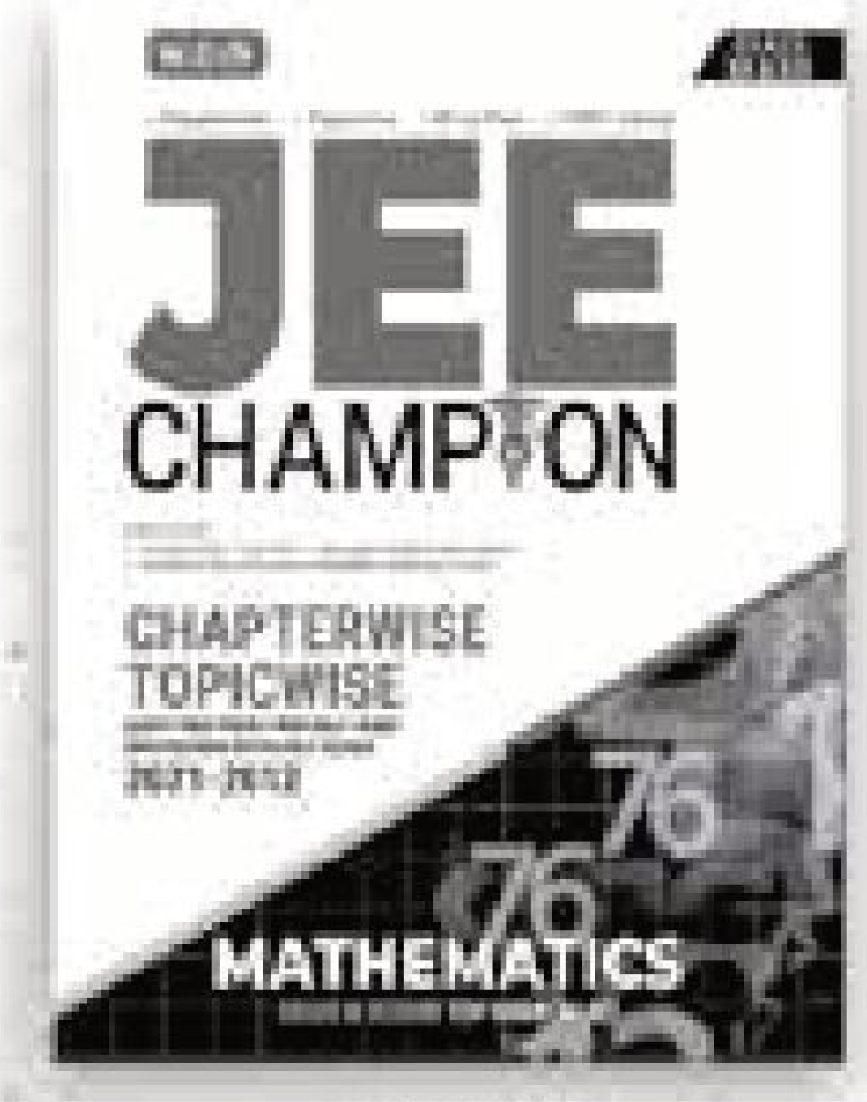
Our Books


JEE
CHAMPION
CHAPTERWISE
TOPICWISE
2021-2012
PHYSICS

₹850/-


JEE
CHAMPION
CHAPTERWISE
TOPICWISE
2021-2012
CHEMISTRY

₹850/-


JEE
CHAMPION
CHAPTERWISE
TOPICWISE
2021-2012
MATHEMATICS

₹850/-

JEE 2022 exams are nearing and all the students must be in the stage of preparation with full dedication and hard work. But have you put your knowledge to test? With our 40 years of experience, we would suggest you to put yourself through MTG's JEE Champion Series so that you can be sure about your level of preparation and find your core strengths and capabilities. The best part is you study at a pace you're comfortable with. Because it's all chapters, topicwise.

PRACTICE PAPER 2022
JEE MAIN

Exam Dates
Session-1
20th to 29th June
Session-2
21st to 30th July

SECTION - A
MULTIPLE CHOICE QUESTIONS

1. Find the vector $\vec{r}$ perpendicular to $\vec{a}=\hat{i}-2\hat{j}+5\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{r}\cdot(2\hat{i}+\hat{j}+\hat{k})+8=0$.

(a) $-13\hat{i}+11\hat{j}+7\hat{k}$ (b) $13\hat{i}-11\hat{j}-7\hat{k}$
(c) $11\hat{i}+13\hat{j}+9\hat{k}$ (d) $7\hat{i}+13\hat{j}+11\hat{k}$

2. If the equation of the parabola is $x^2 = -8y$, the equation of the directix and length of latus rectum, respectively are

(a) $y = 3, 4$ (b) $y = -2, 4$
(c) $y = 2, 8$ (d) $y = -2, 8$

3. If 1, $\log_{81}(3^x + 48)$, $\log_9\left(3^x-\frac{8}{3}\right)$ are in A.P., then the value of x equals

(a) 9 (b) 6 (c) 2 (d) 4

4. If $f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x}$, $x \neq 0$ & $S = \{x \in R : f(-x) = f(x)\}$, then S contains

(a) 4 elements (b) 1 element
(c) 2 elements (d) 5 elements

5. The equation of the tangent to $4x^2 - 9y^2 = 36$ which is perpendicular to the straight line $5x + 2y - 10 = 0$ is

(a) $5(y-3)=4\left(x-\frac{\sqrt{11}}{2}\right)$
(b) $2x-5y+10-12\sqrt{3}=0$
(c) $2x-5y-10+12\sqrt{3}=0$
(d) None of these

6. The position vector of a point P is $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, where $x, y, z \in N$ and $\vec{a}=\hat{i}+\hat{j}+\hat{k}$. If $\vec{r}\cdot\vec{a} = 12$, then number of possible positions of P are

(a) 45 (b) 55
(c) 35 (d) None of these

7. $\lim_{x\to 0}\left(\frac{x}{\sqrt{1+x}-\sqrt{1-x}}\right)$ is equal to

(a) 0 (b) 1 (c) 2 (d) – 1

8. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function satisfying $f(x) + g(x) = x^2$, then the value of $\int_0^1 f(x)g(x)dx$ equals

(a) $\frac{e-3}{2}$ (b) $\frac{e^2-3}{2}$
(c) $\frac{e^2-2}{2}$ (d) None of these

9. Evaluate : $\int\sqrt[3]{\frac{\sin^2 x}{\cos^{14} x}}dx$

(a) $-\frac{3}{5}\tan^{5/3}x-\frac{3}{11}\tan^{-11/3}+C$
(b) $\frac{3}{5}\tan^{5/3}x+\frac{3}{11}\tan^{11/3}x+C$
(c) $\frac{3}{5}\tan^{5/3}x-\frac{3}{11}\tan^{11/3}x+C$
(d) $\frac{3}{5}\tan^{3/5}x+\frac{3}{11}\tan^{3/11}x+C$

10. If the angles of a triangle are in the ratio 2 : 3 : 7, then the sides opposite to the angles are in the ratio

(a) $\sqrt{2}:2:\sqrt{3}+1$ (b) $2:\sqrt{2}:\sqrt{3}+1$
(c) $\frac{1}{2}:\sqrt{2}:\frac{2}{\sqrt{3}-1}$ (d) $\frac{1}{\sqrt{2}}:\frac{1}{2}:\frac{\sqrt{3}+1}{2}$

11. The value of $\int_0^{2\pi}\frac{x\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}dx\ (n\in N)$ is

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) π^2 (d) $2\pi^2$

12. The equation of the plane through the line of intersection of $\vec{r}\cdot(\hat{i}-2\hat{j}+2\hat{k}) = 1$ and $\vec{r}\cdot(2\hat{i}-\hat{j}) + 2 = 0$ and perpendicular to $\vec{r}\cdot(\hat{i}+\hat{j}+2\hat{k}) + 9 = 0$ is

(a) $\vec{r}\cdot(-5\hat{i}+\hat{j}+2\hat{k}) = 7$ (b) $\vec{r}\cdot(5\hat{i}+\hat{j}+2\hat{k}) = 7$
(c) $\vec{r}\cdot(5\hat{i}-\hat{j}-2\hat{k})=7$ (d) None of these

13. Let $f(x) = x^2 - x + 1,\ x \ge \frac{1}{2}$, then the solution of the equation $f(x) = f^{-1}(x)$ is
(a) $x = 1$ (b) $x = 2$
(c) $x = \frac{1}{2}$ (d) None of these

14. Find the equation of the hyperbola whose vertices are $(\pm 6, 0)$ and one of the directrices is $x = 4$.
(a) $\frac{x^2}{6} - \frac{y^2}{5} = 1$ (b) $\frac{x^2}{36} - \frac{y^2}{45} = 1$
(c) $\frac{y^2}{36} + \frac{x^2}{45} = 1$ (d) $\frac{x^2}{45} - \frac{y^2}{36} = 1$

15. Number of solutions of the equation $|z|^2 + 7\bar{z} = 0$ is
(a) 1 (b) 2 (c) 4 (d) 6

16. Ratio in which the *xy*-plane divides the join of (1, 2, 3) and (4, 2, 1) is
(a) 3 : 1 internally (b) 3 : 1 externally
(c) 2 : 1 internally (d) 2 : 1 externally

17. If $y = \frac{ax - b}{(x-1)(x-4)}$ has a turning point $P(2, -1)$, then find the value of a and b respectively.
(a) 1, 2 (b) 2, 1 (c) 0, 1 (d) 1, 0

18. The area of the region in the first quadrant enclosed by x-axis, line $x = \sqrt{3}y$ and the circle $x^2 + y^2 = 4$ is
(a) $\frac{\pi}{3}$ sq. units (b) $\frac{\pi}{6}$ sq. units
(c) $\frac{\pi}{2}$ sq. units (d) None of these

19. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then find $(AB)^{-1}$.
(a) $\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$ (b) $\frac{1}{11}\begin{bmatrix} 14 & -5 \\ -5 & 1 \end{bmatrix}$
(c) $\frac{1}{11}\begin{bmatrix} 1 & 5 \\ 5 & 14 \end{bmatrix}$ (d) $\frac{1}{11}\begin{bmatrix} 1 & -5 \\ -5 & 14 \end{bmatrix}$

20. Find the coefficient of x^{11} in the expansion of $(1 - 2x + 3x^2)(1 + x)^{11}$.
(a) 132 (b) 169 (c) 144 (d) 184

SECTION - B

NUMERICAL TYPE

Attempt any 5 questions out of 10.

21. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2y^6$ is $\frac{1}{x^5y^5} = \frac{p}{qx^2} + c$, then the value of $p + q$ is ________ .

22. The number of real roots of the equation $(15 + 4\sqrt{14})^t + (15 - 4\sqrt{14})^t = 30$, where $t = x^2 - 4|x|$ is _______ .

23. A peon has eight master keys to open several new rooms. Only one master key will open any given room. If 40% of these rooms are usually left unlocked, then the probability that the peon gets into specific room if he selects three master keys at random is $\frac{5}{p}$, where p is _______.

24. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is ________.

25. If $\log_{10} x + \frac{1}{2}\log_{10} x + \frac{1}{4}\log_{10} x + \ldots = y$ and $\frac{1 + 3 + 5 + \ldots + (2y - 1)}{4 + 7 + 10 + \ldots + (3y + 1)} = \frac{20}{7\log_{10} x}$ $\forall x, y \in N$, then the value of $(\log_y x)^5$ must be _______.

26. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$ and I is the unit matrix of order 3, then $A^2 + 2A^4 + 4A^6$ is equal to kA^8. The value of k is ________.

27. Seven digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a random order. The probability that this seven digit number is divisible by 9 is p, then the value of $18p$ is _____.

28. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2 & , x < 0 \\ 0 & , x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2 & , x > 0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is _________.

29. Set A has m elements and Set B has n elements. If the total number of subsets of A is 112 more than the total number of subsets of B, then the value of $m \cdot n$ is _______.

30. The number of integral values of k for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is ________.

SOLUTIONS

1. (a) : Since $\vec{r}$ is perpendicular to both $\vec{a}$ and $\vec{b}$

$\therefore \quad \vec{r} = \lambda(\vec{a} \times \vec{b})$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 5 \\ 2 & 3 & -1 \end{vmatrix} = \lambda(-13\hat{i} + 11\hat{j} + 7\hat{k})$$

Now, $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) + 8 = 0$

$\Rightarrow \; -8\lambda + 8 = 0 \Rightarrow \lambda = 1$

$\therefore \quad \vec{r} = -13\hat{i} + 11\hat{j} + 7\hat{k}$

2. (c) : Comparing the given equation with standard form $x^2 = -4ay$, we get $a = 2$.

Therefore, the equation of directrix is $y = 2$ and the length of the latus rectum is $4a$, *i.e.*, 8.

3. (c) : Given, $1, \log_9^2(3^x + 48), \log_9(3^x - 8/3) \in$ A.P.

$\Rightarrow \; \log_9 9, \dfrac{1}{2}\log_9(3^x + 48), \log_9(3^x - 8/3) \in$ A.P.

$\Rightarrow \; 9, (3^x + 48)^{1/2}, 3^x - 8/3 \in$ G.P.

[$\because$ $\log a, \log b, \log c \in$ A.P. then $a, b, c \in$ G.P.]

$\Rightarrow \; 3^x + 48 = 9(3^x - 8/3)$

$\Rightarrow \; 8 \cdot 3^x = 72 \Rightarrow 3^x = 9 \Rightarrow 3^x = 3^2 \Rightarrow x = 2.$

4. (c) : Given, $f\left(\dfrac{1}{x}\right) + 2f(x) = 3\left(\dfrac{1}{x}\right)$ ...(i)

$\Rightarrow \; f(x) + 2f\left(\dfrac{1}{x}\right) = 3x$ ...(ii)

(Replacing $x \to \dfrac{1}{x}$)

Now (ii) – 2 × (i), we have

$$-3f(x) = 3x - \frac{6}{x} \Rightarrow f(x) = \frac{2}{x} - x$$

Now for S, we have $f(-x) = f(x)$ *i.e.*, $f(x) = f(-x)$

$\Rightarrow \; \dfrac{2}{x} - x = \dfrac{-2}{x} + x$

$\Rightarrow \; \dfrac{4}{x} = 2x \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

$\therefore$ S contains exactly two elements.

5. (d): We have, $4x^2 - 9y^2 = 36$ ...(i)

$\Rightarrow \; 8x - 18y\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{4x}{9y}$

$\therefore$ Slope of the tangent $= \dfrac{4x}{9y}$

Also, slope of line $5x + 2y - 10 = 0$ is $\dfrac{-5}{2}$

$\because$ Line is perpendicular to the tangent, so the product of slopes = – 1

$\therefore \quad \dfrac{4x}{9y} \times \left(-\dfrac{5}{2}\right) = -1 \Rightarrow y = \dfrac{10x}{9}$ ...(ii)

Using (ii) in (i), we get

$$4x^2 - \frac{100x^2}{9} = 36 \Rightarrow -64x^2 = 324,$$

which gives imaginary x.

Hence, there is no point on the curve at which tangent is perpendicular to the given line.

6. (b): Given, $\vec{r} \cdot \vec{a} = 12$

$\Rightarrow \; x + y + z = 12$ and $x, y, z \in N$

$\therefore$ Required number of positions of P

= Number of positive integral solutions of the equation $x + y + z = 12$

= Coefficient of x^{12} in $(x + x^2 + x^3 + \ldots)^3$

= Coefficient of x^{12} in $x^3(1 + x + x^2 + \ldots)^3$

= Coefficient of x^9 in $((1 - x)^{-1})^3$

= Coefficient of x^9 in $(1 - x)^{-3}$

= Coefficient of x^9 in $(1 + {}^3C_1x + {}^4C_2x^2 + \ldots + {}^{11}C_9x^9 + \ldots)$

$= {}^{11}C_9 = {}^{11}C_2 = 55$

7. (b): We have, $\lim\limits_{x \to 0}\left(\dfrac{x}{\sqrt{1+x} - \sqrt{1-x}}\right)$

$$= \lim_{x \to 0}\left\{\frac{x(\sqrt{1+x} + \sqrt{1-x})}{(1+x) - (1-x)}\right\} = \lim_{x \to 0}\left\{\frac{x(\sqrt{1+x} + \sqrt{1-x})}{2x}\right\}$$

$$= \frac{1}{2}\lim_{x \to 0}\{\sqrt{1+x} + \sqrt{1-x}\} = \frac{1}{2} \times 2 = 1$$

8. (d): Given, $f'(x) = f(x)$

$\Rightarrow \; \dfrac{f'(x)}{f(x)} = 1 \Rightarrow \log f(x) = x + \log c \Rightarrow f(x) = ce^x$

Now, $f(0) = 1 \quad \therefore \; c = 1$

$$\therefore \quad \int_0^1 f(x)g(x)dx = \int_0^1 e^x\left(x^2 - e^x\right)dx$$

$$= \int_0^1 \left(x^2e^x - e^{2x}\right)dx = \left[e^x\left(x^2 - 2x + 2\right) - \frac{e^{2x}}{2}\right]_0^1 = e - \frac{e^2}{2} - \frac{3}{2}$$

9. (b): Let $I = \displaystyle\int \sqrt[3]{\frac{\sin^2 x}{\cos^{14} x}}dx = \int \sin^{2/3} x \cos^{-14/3} x \, dx$

Here, the sum of the exponents of $\sin x$ and $\cos x$ is –4, which is a negative even integer. So, we divide and multiply I by $\cos^4 x$, we get

$$\therefore \quad I = \int \sin^{2/3} x \cos^{-14/3} x \cos^4 x \sec^4 x \, dx$$

$$= \int \frac{\sin^{2/3} x}{\cos^{2/3} x}\sec^4 x \, dx = \int \tan^{2/3} x (1 + \tan^2 x)\sec^2 x \, dx$$

Putting $\tan x = t \Rightarrow \sec^2 x \, dx = dt$, we get

$$I = \int (t^{2/3} + t^{8/3})dt = \frac{3}{5}t^{5/3} + \frac{3}{11}t^{11/3} + C$$

$$= \frac{3}{5}\tan^{5/3} x + \frac{3}{11}\tan^{11/3} x + C$$

10. (a) : Given, angles are in ratio 2 : 3 : 7

Then, $2X + 3X + 7X = 180°$

$\Rightarrow\ X = 15°$

$\therefore$ Angles are 30°, 45°, 105°

Let $A : B : C = 30° : 45° : 105°$

Then, $\sin A : \sin B : \sin C = \sin 30° : \sin 45° : \sin 105°$

$= \sin 30° : \sin 45° : \sin (45° + 60°)$

$$= \frac{1}{2} : \frac{1}{\sqrt{2}} : \frac{(\sqrt{3}+1)}{2\sqrt{2}} = \sqrt{2} : 2 : (\sqrt{3}+1)$$

Using sine formula, we get $a : b : c = \sqrt{2} : 2 : (\sqrt{3}+1)$

11. (c) : Let $I = \int\limits_0^{2\pi} \frac{x\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}dx$ ...(i)

$$\Rightarrow\ I = \int\limits_0^{2\pi} \frac{(2\pi - x)\sin^{2n}(2\pi - x)}{\sin^{2n}(2\pi - x) + \cos^{2n}(2\pi - x)}dx$$

$$\Rightarrow\ I = \int\limits_0^{2\pi} \frac{(2\pi - x)\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = 2\pi \int\limits_0^{2\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}dx$$

$$\therefore\ I = \pi \int\limits_0^{2\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}dx = \pi \cdot 2\int\limits_0^{\pi} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}dx$$

$$\therefore\ I = 4\pi \int\limits_0^{\pi/2} \frac{\sin^{2n} x}{\sin^{2n} x + \cos^{2n} x}dx \quad \text{...(iii)}$$

$$= 4\pi \int\limits_0^{\pi/2} \frac{\sin^{2n}\left(\frac{\pi}{2} - x\right)}{\sin^{2n}\left(\frac{\pi}{2} - x\right) + \cos^{2n}\left(\frac{\pi}{2} - x\right)}dx$$

$$\Rightarrow\ I = 4\pi \int\limits_0^{\pi/2} \frac{\cos^{2n} x}{\cos^{2n} x + \sin^{2n} x}dx \quad \text{...(iv)}$$

Adding (iii) and (iv), we get

$$2I = 4\pi \int\limits_0^{\pi/2} 1 \cdot dx = 4\pi \cdot \frac{\pi}{2} \qquad \therefore\ I = \pi^2$$

12. (a) : Equation of any plane through the intersection of given planes is

$$\left[\vec{r} \cdot (\hat{i} - 2\hat{j} + 2\hat{k}) - 1\right] + \lambda\, [\vec{r} \cdot (2\hat{i} - \hat{j}) + 2] = 0$$

$$\Rightarrow\ \left[\vec{r} \cdot ((1+2\lambda)\hat{i} + (-2-\lambda)\hat{j} + 2\hat{k})\right] = 1 - 2\lambda \quad \text{...(i)}$$

Now, (i) will be $\perp$ to $\vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) + 9 = 0$

$\Rightarrow\ 1(1 + 2\lambda) + (-2 - \lambda)(1) + 2 \cdot 2 = 0$

$\Rightarrow\ (2\lambda - \lambda) + 1 - 2 + 4 = 0 \quad \Rightarrow\ \lambda = -3$

$\therefore$ Required equation of plane is

$\vec{r} \cdot (-5\hat{i} + \hat{j} + 2\hat{k}) = 7$

13. (a) : Let $y = x^2 - x + 1$

$$\Rightarrow\ x = \frac{1 \pm \sqrt{1 - 4(1 - y)}}{2}$$

$$\because\ x \geq \frac{1}{2} \quad \therefore\ x = \frac{1}{2} + \sqrt{y - \frac{3}{4}}$$

$$\Rightarrow\ f^{-1}(x) = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$$

Now, $x^2 - x + 1 = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$

$\Rightarrow\ x = 1$ (By checking the given possibilities)

14. (b): As the vertices are on the x-axis, the equation is of the type $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since $b^2 = a^2(e^2 - 1)$, vertices are $(\pm a, 0)$ and directrices are given by $x = \pm\frac{a}{e}$.

$\therefore\ a = 6$, $\frac{a}{e} = 4$ and so $e = \frac{3}{2}$, which gives $b^2 = 36\left(\frac{9}{4} - 1\right) = 45$.

$\therefore$ The required equation of the hyperbola is $\frac{x^2}{36} - \frac{y^2}{45} = 1$.

15. (b): Given, $|z|^2 + 7\bar{z} = 0$

$\Rightarrow\ z\bar{z} + 7\bar{z} = 0 \Rightarrow\ \bar{z}\,(z + 7) = 0$

Case (i) : $\bar{z} = 0$, $\therefore\ z = 0 = 0 + i0$

Case (ii) : $z = -7$ $\therefore\ z = -7 + 0i$

Hence there are only two solutions.

$z = 0$ and $z = -7$

16. (b): Suppose xy-plane divides the join of (1, 2, 3) and (4, 2, 1) in the ratio $\lambda : 1$. Then the coordinates of the point of division are $\left(\frac{4\lambda+1}{\lambda+1}, \frac{2\lambda+2}{\lambda+1}, \frac{\lambda+3}{\lambda+1}\right)$. This point lies on xy-plane. So, its z-coordinate = 0

$$\Rightarrow\ \frac{\lambda+3}{\lambda+1} = 0 \Rightarrow\ \lambda = -3$$

Hence, xy-plane divides the join of (1, 2, 3) and (4, 2, 1) externally in the ratio 3 : 1.

17. (d): We have, $y = \frac{ax - b}{(x-1)(x-4)} = \frac{ax - b}{x^2 - 5x + 4}$ ...(i)

$$\Rightarrow\ \frac{dy}{dx} = \frac{(x^2 - 5x + 4)a - (ax - b)(2x - 5)}{(x^2 - 5x + 4)^2} \quad \text{...(ii)}$$

$$\Rightarrow\ \left(\frac{dy}{dx}\right)_{P(2,-1)} = \frac{(4 - 10 + 4)a - (2a - b)(4 - 5)}{(4 - 10 + 4)^2} = -\frac{b}{4}$$

Since P is a turning point of the curve (i). Therefore,

$$\Rightarrow\ \left(\frac{dy}{dx}\right)_{P(2,-1)} = 0 \Rightarrow\ -\frac{b}{4} = 0 \Rightarrow\ b = 0 \quad \text{...(iii)}$$

Since $P(2, -1)$ lies on $y = \dfrac{ax - b}{(x-1)(x-4)}$

$$-1 = \frac{2a-b}{(2-1)(2-4)} \Rightarrow -1 = \frac{2a-b}{-2} \Rightarrow 2a-b=2 \quad \text{...(iv)}$$

From (iii) and (iv), we get $a = 1$, $b = 0$

18. (a) : The given equations are

$x = \sqrt{3}y$ ...(1)

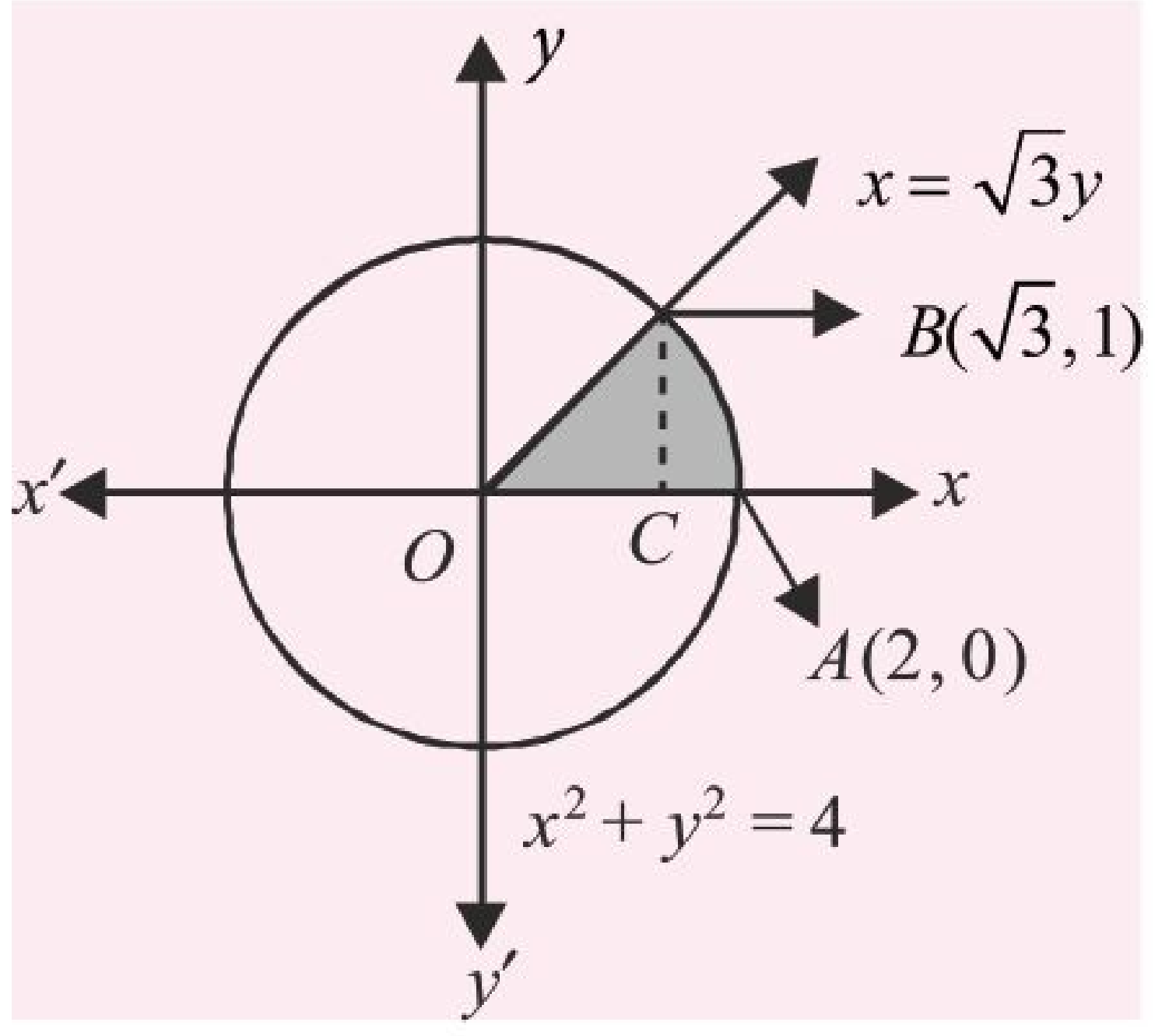
and $x^2 + y^2 = 4$ ...(2)

From (1), we get $y = \dfrac{x}{\sqrt{3}}$

Putting $y = \dfrac{x}{\sqrt{3}}$ in (2),

we get

$$x^2 + \frac{x^2}{3} = 4 \Rightarrow \frac{4x^2}{3} = 4 \Rightarrow \quad x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$



When $x = \pm\sqrt{3}, y = \pm 1$

Thus B is $(\sqrt{3}, 1)$ (Required region is in first quadrant)

$\therefore$ Required area = area (OBC) + area (BAC)

$$= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}}dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$$

$$= \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} x\,dx + \int_{\sqrt{3}}^{2} \sqrt{2^2-x^2}\,dx$$

$$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}} + \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{2\sqrt{3}}[3-0] + \left[\left\{0 + 2\sin^{-1}(1)\right\} - \left\{\frac{\sqrt{3}(1)}{2} + 2\sin^{-1}\frac{\sqrt{3}}{2}\right\}\right]$$

$$= \frac{\sqrt{3}}{2} + 2\left(\frac{\pi}{2}\right) - \frac{\sqrt{3}}{2} - 2\left(\frac{\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq.units}$$

19. (a) : Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$

$\therefore \quad |AB| = \begin{vmatrix} -1 & 5 \\ 5 & -14 \end{vmatrix} = 14 - 25 = -11 \neq 0$

$\therefore \quad (AB)^{-1}$ exists. $\therefore \quad (AB)^{-1} = \dfrac{1}{|AB|}$ [adj (AB)]

$$= \frac{-1}{11}\begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$

20. (c) : We have, $(1 - 2x + 3x^2)(1 + x)^{11}$

$= (1 - 2x + 3x^2) \times (1 + {}^{11}C_1x + {}^{11}C_2x^2 + ... + {}^{11}C_9x^9 + {}^{11}C_{10}x^{10} + x^{11})$

$\therefore$ Coefficient of $x^{11} = 1 \times 1 - 2 \times {}^{11}C_{10} + 3 \times {}^{11}C_9$

$$= 1 - 2\times 11 + 3\times\frac{11\times 10}{2\times 1} = 144$$

21. (7) : $\dfrac{dy}{dx} + \dfrac{y}{x} = x^2y^6 \Rightarrow \dfrac{1}{y^6}\dfrac{dy}{dx} + \dfrac{1}{xy^5} = x^2$

Put $\dfrac{1}{y^5} = z \Rightarrow \dfrac{-5}{y^6}\dfrac{dy}{dx} = \dfrac{dz}{dx}$

$\therefore \quad -\dfrac{1}{5}\left(\dfrac{dz}{dx}\right) + \dfrac{z}{x} = x^2 \Rightarrow \dfrac{dz}{dx} - \dfrac{5}{x}z = -5x^2$, which is a linear differential equation

$$\therefore \quad \text{I.F.} = e^{-5\int\frac{dx}{x}} = e^{-5\log x} = \frac{1}{x^5}$$

Hence, solution is $z\cdot\dfrac{1}{x^5} = -5\displaystyle\int\frac{dx}{x^3} + c \Rightarrow \dfrac{1}{x^5y^5} = \dfrac{5}{2x^2} + c$

$\therefore \quad p = 5$ and $q = 2$

$\Rightarrow \quad p + q = 5 + 2 = 7$

22. (6) : $15 - 4\sqrt{14} = \left(15 + 4\sqrt{14}\right)^{-1}$

If $y = \left(15 + 4\sqrt{14}\right)^t$, then $y + \dfrac{1}{y} = 30$

$\Rightarrow \quad y^2 - 30y + 1 = 0$

$\Rightarrow \quad y = 15 \pm 4\sqrt{14} = \left(15 + 4\sqrt{14}\right)^t$

$\therefore \quad t = \pm 1 \Rightarrow x^2 - 4|x| = \pm 1$

$(|x| - 2)^2 = 3, 5 \Rightarrow |x| = 2 \pm \sqrt{3}, 2 + \sqrt{5}$

$x = \pm\sqrt{2-\sqrt{3}}, \pm\sqrt{2+\sqrt{3}}, \pm\sqrt{2+\sqrt{5}}.$

23. (8) : Let A_1 and A_2 denote the event that a specific room is unlocked and locked respectively.

$\therefore \quad P(A_1) = 0.4, P(A_2) = 0.6$

If A denotes the event that the peon gets into a specific room, then

$$P(A|A_1) = 1, P(A|A_2) = \frac{{}^1C_1 \times {}^7C_2}{{}^8C_3} = \frac{3}{8}$$

$\therefore \quad P(A) = P(A_1)\,P(A|A_1) + P(A_2)\,P(A|A_2)$

$$= (0.4)(1) + (0.6)\left(\frac{3}{8}\right) = \frac{4}{10} + \frac{18}{80} = \frac{5}{8}.$$

$\therefore \quad p = 8.$

24. (9) : Let $y = \dfrac{4}{\sin x} + \dfrac{1}{1 - \sin x} = \dfrac{4 - 3\sin x}{\sin x(1 - \sin x)}$

Let $\sin x = t$, where $t \in (0, 1)$

$\therefore \quad y = \dfrac{4 - 3t}{t - t^2}.$

Now, $\frac{dy}{dt}=\frac{(t-t^2)(-3)-(4-3t)(1-2t)}{(t-t^2)^2}=0$

$\Rightarrow\ 3t^2-3t-(4-11t+6t^2)=0$

$\Rightarrow\ 3t^2-8t+4=0 \Rightarrow (3t-2)(t-2)=0$

$\Rightarrow\ t=\frac{2}{3}\quad [\because\ t\in(0,1)] \ \Rightarrow \sin x=\frac{2}{3}$

$\therefore\ y_{\min}=\frac{4}{2/3}+\frac{1}{1-2/3}=9 \qquad [\because\ y_{\max}\to\infty]$

$\Rightarrow\ \alpha_{\min}=9$

25. (3125) : From the first equation, we have

$$\log_{10}x\left\{1+\frac{1}{2}+\frac{1}{4}+\ldots\infty\right\}=y$$

$$\Rightarrow\ \log_{10}x\left\{\frac{1}{1-\frac{1}{2}}\right\}=y$$

$$\Rightarrow\ 2\log_{10}x=y \qquad \ldots\text{(i)}$$

From the second equation, we have

$$\frac{1+3+5+\ldots+(2y-1)}{4+7+10+\ldots+(3y+1)}=\frac{20}{7\log_{10}x}$$

$$\Rightarrow\ \frac{\frac{y}{2}\{1+2y-1\}}{\frac{y}{2}\{4+3y+1\}}=\frac{20}{7\left(\frac{y}{2}\right)} \qquad \text{[From (i)]}$$

$$\Rightarrow\ \frac{2y}{3y+5}=\frac{40}{7y}\Rightarrow 7y^2=60y+100$$

$\Rightarrow\ 7y^2-60y-100=0$

$\Rightarrow\ (y-10)(7y+10)=0$

$\therefore\ y=10 \qquad (\because\ y\neq-\frac{10}{7}$ as $y\in N)$

Now, from (i), $2\log_{10}x=10$

$\Rightarrow\ \log_{10}x=5\ \Rightarrow\ x=10^5$

Now, $\log_y x=\log_{10}10^5=5$

$\therefore\ (\log_y x)^5=5^5=3125$

26. (7) : $A^2=\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$A^2=A^4=A^6=A^8=I_3$

$\therefore\ A^2+2A^4+4A^6$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}+\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}$$

$$=\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}=7I_3=7A^8=kA^8 \quad \therefore\ k=7$$

27. (2): Given digits are 1, 2, 3, 4, 5, 6, 7, 8, 9.

Total number of 7 digit numbers = ${}^9C_7\times 7!$

For divisibility by 9, sum of the digits must be divisible by 9.

$\therefore$ We have to include the digit 9 and 3 pairs out of the followings : (1, 8), (2, 7), (3, 6), (4, 5)

Favourable number of ways = ${}^4C_3\,.\,7!$

$\therefore$ Required probability = $\frac{{}^4C_3\cdot 7!}{{}^9C_7\cdot 7!}=\frac{4}{36}=\frac{1}{9}$

28. (5): We have,

$$f(x)=\begin{cases}x^5\sin\left(\frac{1}{x}\right)+5x^2, & x<0\\ 0, & x=0\\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2, & x>0\end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}5x^4\sin\left(\frac{1}{x}\right)+x^5\cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)+10x, & x<0\\ 0, & x=0\\ 5x^4\cos\left(\frac{1}{x}\right)+x^5\left(-\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\right)+2\lambda x, & x>0\end{cases}$$

$$\Rightarrow\ f'(x)=\begin{cases}5x^4\sin\left(\frac{1}{x}\right)-x^3\cos\left(\frac{1}{x}\right)+10x, & x<0\\ 0, & x=0\\ 5x^4\cos\left(\frac{1}{x}\right)+x^3\sin\left(\frac{1}{x}\right)+2\lambda x, & x>0\end{cases}$$

$$\Rightarrow\ f''(x)=\begin{cases}[20x^3\sin\left(\frac{1}{x}\right)-5x^2\cos\left(\frac{1}{x}\right)\\ \quad -3x^2\cos\left(\frac{1}{x}\right)-x\sin\left(\frac{1}{x}\right)+10], & x<0\\ 0, & x=0\\ [20x^3\cos\left(\frac{1}{x}\right)+5x^2\sin\left(\frac{1}{x}\right)\\ \quad +3x^2\sin\left(\frac{1}{x}\right)-x\cos\left(\frac{1}{x}\right)+2\lambda], & x>0\end{cases}$$

To exist $f''(0)$, $f''(0^+)=f''(0^-)$

$\Rightarrow\ 2\lambda=10\ \Rightarrow\ \lambda=5$

29. (28) : We know that number of subsets of a set containing n elements is 2^n.

According to question,

$2^m=2^n+112$

$\Rightarrow\ 2^m-2^n=112\Rightarrow 2^n(2^{m-n}-1)=16\times 7=2^4\times(8-1)$

$\Rightarrow\ 2^n(2^{m-n}-1)=2^4(2^3-1)$

Thus, $m-n=3$ and $n=4\ \Rightarrow\ m=7$

So, $m\times n=7\times 4=28$

30. (11) : We have, $3\sin x+4\cos x=k+1$

$\Rightarrow\ (k+1)\in\left[-\sqrt{3^2+4^2},\ \sqrt{3^2+4^2}\right]$

$\Rightarrow\ (k+1)\in[-5,5]\ \Rightarrow k\in[-6,4]$

$\therefore$ Number of integral values of k is 11.

◆◆

GEAR UP
for JEE MAIN 2022

Exam Dates
Session-1
20th to 29th June
Session-2
21st to 30th July

Section A will be of Multiple Choice Questions (MCQs). Section B will contain questions whose answers are to be filled in as a Numerical Value. In Section B candidates have to attempt any five questions out of 10.

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The area of the region bounded by the curves $y = x^2$, $y = |2 - x^2|$ and $y = 2$ which lies to the right of the line $x = 1$ is

(a) $\left(\frac{12-20\sqrt{3}}{2}\right)$ sq. units

(b) $\left(\frac{20-\sqrt{2}}{3}\right)$ sq. units

(c) $\left(\frac{20-12\sqrt{2}}{3}\right)$ sq. units

(d) $\left(\frac{12-20\sqrt{2}}{3}\right)$ sq. units

2. The function $f : R \rightarrow R$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

3. If $x = \cos\theta$ and $y = \sin 5\theta$, then $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} =$

(a) $-5y$ (b) $5y$ (c) 25 (d) $-25y$

4. Points at which the curves $\arg(z-3i) = \frac{3\pi}{4}$ and $\arg(2z-1-2i) = \frac{\pi}{4}$ intersect, is

(a) $\left(\frac{3}{4}, \frac{9}{4}\right)$ (b) $\left(\frac{5}{4}, \frac{7}{4}\right)$

(c) $\left(\frac{2}{4}, \frac{7}{4}\right)$ (d) $\left(\frac{6}{7}, \frac{4}{7}\right)$

5. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S

(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements.

6. The tangent at a point $P(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle at two points. The chord joining them subtends a right angle at the centre. Then, the eccentricity of the ellipse is given by

(a) $(1 + \sin^2\theta)^{-1/2}$ (b) $(1 - \cos^2\theta)^{-1/2}$
(c) $(1 + \sin^2\theta)$ (d) $(1 + \cos^2\theta)^{1/2}$

7. Let r be the range of $n(\forall\ n \geq 1)$ observations $x_1, x_2, ..., x_n$, if $S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$, then

(a) $S < r\sqrt{\frac{n^2+1}{n-1}}$ (b) $S \geq r\sqrt{\frac{n}{n-1}}$

(c) $S = r\sqrt{\frac{n}{n-1}}$ (d) $S < r\sqrt{\frac{n}{n-1}}$

8. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = \hat{j} - \hat{k}$. If $\vec{b}$ is a vector satisfying $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$, then $\vec{b}$ is

(a) $\frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$ (b) $\frac{1}{3}(5\hat{i} - 2\hat{j} - 2\hat{k})$

(c) $3\hat{i} - \hat{j} - \hat{k}$ (d) none of these

9. The coordinates of the foot of the perpendicular drawn from the point $A(2, 4, -1)$ on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ are

(a) $(1, -3, 4)$ (b) $(-4, 1, -3)$
(c) $(4, 1, 3)$ (d) none of these

10. The value of a so that the volume of the parallelepiped formed by $\hat{i}+a\hat{j}+\hat{k}$, $\hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum is

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

11. The probability that a scheduled flight departs on time is 0.9, the probability that it arrives on time is 0.8 and the probability that it departs and arrives on time is 0.7. Then the probability that a plane arrives on time, given that it departs on time, is

(a) 0.72 (b) 8/9 (c) 7/9 (d) 0.56

12. If $A=\begin{bmatrix}0 & 1 & -1\\ 2 & 1 & 3\\ 3 & 2 & 1\end{bmatrix}$, then $(A(\text{adj}A)A^{-1})A=$

(a) $2\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$ (b) $\begin{bmatrix}-6 & 0 & 0\\ 0 & -6 & 0\\ 0 & 0 & -6\end{bmatrix}$

(c) $\frac{1}{6}\begin{bmatrix}0 & 1 & -1\\ 2 & 1 & 3\\ 3 & 2 & 1\end{bmatrix}$ (d) none of these

13. If the system of linear equations

$x + 4ay + az = 0$
$x + 3by + bz = 0$
$x + 2cy + cz = 0$

have a non-trivial solution, then a, b, c are in

(a) H.P. (b) G.P.
(c) A.P. (d) none of these

14. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x}+\sec\left(\frac{y}{x}\right)$, $x > 0$. Then, the equation of the curve is

(a) $\sin\left(\frac{y}{x}\right)=\log x+\frac{1}{2}$ (b) $\operatorname{cosec}\left(\frac{y}{x}\right)=\log x+2$

(c) $\sec\left(\frac{y}{x}\right)=\log x+2$ (d) $\cos\left(\frac{2y}{x}\right)=\log x+\frac{1}{2}$

15. If $h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$ and $f'(x) > 0\ \forall\ x$, then

(a) $h(x)$ is an increasing function of x in some specified interval
(b) $h(x)$ is an increasing function $\forall\ x \in R$
(c) $h(x)$ is a decreasing function $\forall\ x \in R$
(d) $h(x)$ is a decreasing function of x in some specified interval.

16. If $f(x)=|x|^{|\tan x|}$, then $f'\left(-\frac{\pi}{6}\right)=$

(a) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{\frac{2\sqrt{3}}{\pi}-\frac{4}{3}\log\frac{6}{\pi}\right\}$

(b) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{-\frac{2\sqrt{3}}{\pi}+\frac{4}{3}\log\frac{6}{\pi}\right\}$

(c) $\left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{\frac{2\sqrt{3}}{\pi}+\frac{4}{3}\log\frac{6}{\pi}\right\}$

(d) none of these

17. The value of the series

$C_0 - C_1\frac{2^3}{2}+C_2\frac{2^6}{3}-C_3\frac{2^9}{4}+\ldots(n+1)$ terms is

(a) $\frac{1-(-7)^{n+1}}{8(n+1)}$ (b) $\frac{1-(-7)^{n+1}}{3(n+1)}$

(c) $\frac{2^n-1}{n+1}$ (d) 0

18. The equation of the circle described on the chord $3x + y + 5 = 0$ of the circle $x^2 + y^2 = 16$ as diameter is

(a) $x^2 + y^2 + 3x + y + 11 = 0$
(b) $x^2 + y^2 - 3x - y - 11 = 0$
(c) $x^2 + y^2 + 3x + y - 11 = 0$
(d) $x^2 + y^2 + 3x - y - 11 = 0$

19. Let the population of rabbits surviving at a time be governed by the differential equation $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200.$ If $p(0) = 100$, then $p(t) =$

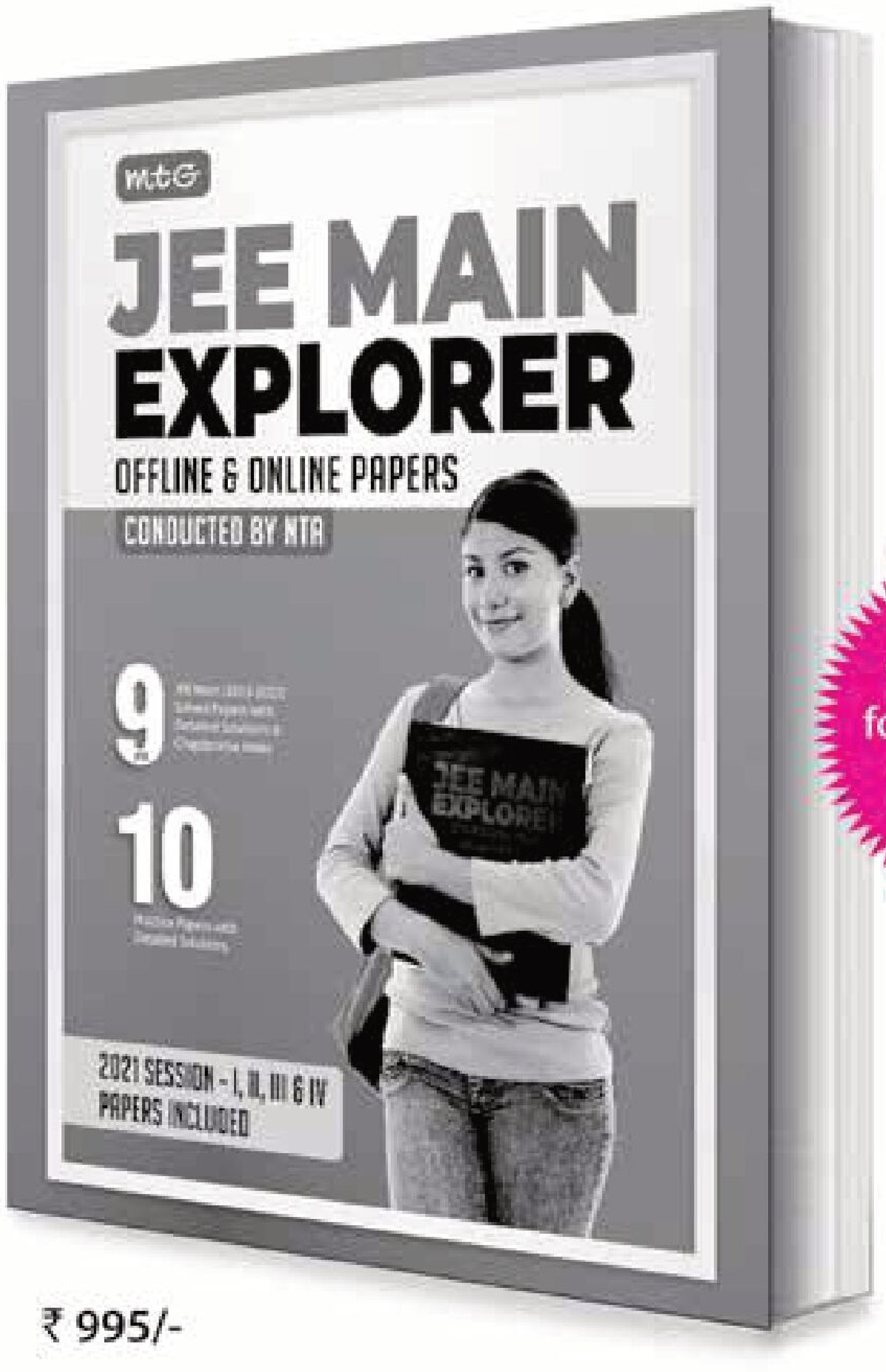
(a) $400 - 300e^{t/2}$ (b) $300 - 200e^{-t/2}$
(c) $600 - 500e^{t/2}$ (d) $400 - 300e^{-t/2}$

20. The equation of the plane passing through the points (0, 1, 0) and (3, 4, 1) and parallel to the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$ is

(a) $4x - 13y + 15z - 13 = 0$
(b) $8x - 13y + 15z + 15 = 0$
(c) $8x - 13y + 15z + 13 = 0$
(d) none of these

mtG

What is JEE (MAIN) like?


mtG
JEE MAIN
EXPLORER
OFFLINE & ONLINE PAPERS
CONDUCTED BY NTA
9
10
2021 SESSION - I, II, III & IV
PAPERS INCLUDED

Visit
www.mtg.in
for latest offers
and to buy
online!

₹ 995/-

Thinking about competing in the JEE? Unsure of what it's like? Get a first-hand feel with MTG's JEE Main Explorer. With solved papers from the past 9 years, glance through the changing pattern of examination or attempt the papers to check your chances of success. Either way, at ₹ 995, isn't this a steal for JEE aspirants like you? So what are you waiting for? **Order MTG's JEE Main Explorer today.**

Scan now with your smartphone or tablet
Application to read QR codes required

Available at all leading book shops throughout the country. To buy online visit www.mtg.in.

For more information or for help in placing your order, call 0124-6601200 or email: *info@mtg.in*

SECTION-B (NUMERICAL VALUE TYPE)

21. The number of solutions of

$$\sin\left\{\sin^{-1}(\log_{1/2} x)\right\}+2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2}-\frac{3}{2}\right)\right\}\right|=0, \text{ is\_\_\_\_.}$$

22. Let a, b, c be the positive integers such that b/a is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2+a-14}{a+1}$ is ________.

23. The value of $3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10°$ is ______.

24. If $\int\frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx=\sin^{-1}\left(\frac{\sin x+\cos x}{a}\right)+c$, then $a =$ ________.

25. If $f(x)=\begin{cases}\dfrac{a|x^2-x-2|}{2+x-x^2}, & x<2\\ b, & x=2\\ \dfrac{x-[x]}{x-2}, & x>2\end{cases}$ is continuous at $x = 2$, then the value of $a + b =$ ______.

26. The four common tangents to the ellipses $\frac{x^2}{9}+\frac{y^2}{4}=1$ and $\frac{x^2}{4}+\frac{y^2}{9}=1$ form a square of area _____ (sq. units).

27. Let $f(x)$ be a continuous in $(-\infty, \infty)$ and $f'(x)$ exists in $(-\infty, \infty)$. If $f(3) = -6$ and $f'(x) \geq 6$ for all $x \in [3, 6]$, then $f(6)$ is greater than or equal to ______.

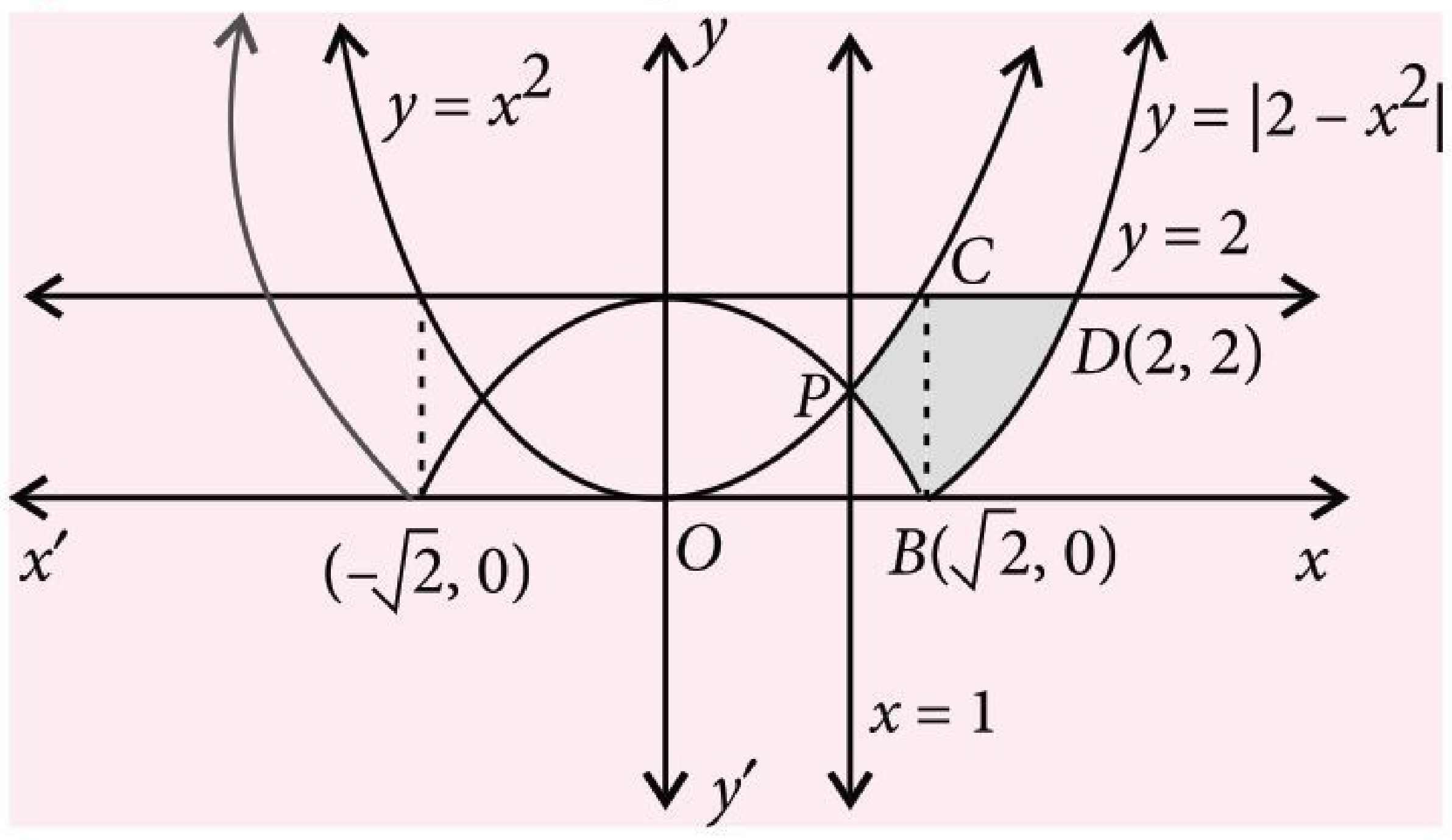
28. The letters of the word 'ARRANGE' is so arranged that two R do not come together. Number of such word is ______.

29. The value of $\int_0^{\pi} e^{\sin^2 x}\cos^3 x\,dx =$ _______.

30. Electric current C, measured by a galvanometer, is given by the equation $C = k\tan\theta$, where k is a constant. Then the percentage error in the current corresponding to an error 0.7 percent in the measurement of θ when $\theta = 45°$ is _______.

SOLUTIONS

1. **(c) :** Let A be the required area.



Then, A = area $PBCP$ + area BCD

$$\Rightarrow\quad A=\int_1^{\sqrt{2}}\{x^2-(2-x^2)\}dx+\int_{\sqrt{2}}^{2}\{2-(x^2-2)\}dx$$

$$\Rightarrow\quad A=\int_1^{\sqrt{2}}(2x^2-2)dx+\int_{\sqrt{2}}^{2}(4-x^2)\,dx$$

$$\Rightarrow\quad A=\left[\frac{2}{3}x^3-2x\right]_1^{\sqrt{2}}+\left[4x-\frac{x^3}{3}\right]_{\sqrt{2}}^{2}$$

$$\Rightarrow\quad A=\left(\frac{20-12\sqrt{2}}{3}\right)\text{sq. units}$$

2. **(b) :** The given function $f: R \to R$ is defined by $f(x) = (x - 1)(x - 2)(x - 3)$

Since, $1, 2, 3 \in R$ and $f(1) = 0, f(2) = 0, f(3) = 0$

Now, $f(1) = f(2)$ but $1 \neq 2$

$\therefore$ f is not a one-one mapping.

Again if $y = (x - 1)(x - 2)(x - 3)$, then $x^3 - 6x^2 + 11x - 6 - y = 0$ and any third degree equation must have at least one real root. Hence it is an onto mapping.

3. **(d) :** We have, $x = \cos\theta$ and $y = \sin 5\theta$

$$\therefore\quad \frac{dx}{d\theta}=-\sin\theta \text{ and } \frac{dy}{d\theta}=5\cos 5\theta$$

$$\Rightarrow\quad \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=-\frac{5\cos 5\theta}{\sin\theta}\qquad\text{...(i)}$$

Now, $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)$

$$=\frac{d}{dx}\left(-\frac{5\cos 5\theta}{\sin\theta}\right)=-5\frac{d}{d\theta}\left(\frac{\cos 5\theta}{\sin\theta}\right)\frac{d\theta}{dx}$$

$$=-5\times\frac{-5\sin 5\theta\sin\theta-\cos\theta\cos 5\theta}{\sin^2\theta}\times\frac{-1}{\sin\theta}$$

$$=\frac{-25\sin 5\theta\sin\theta-5\cos\theta\cos 5\theta}{\sin^3\theta}\qquad\text{...(ii)}$$

Thus, from (i) and (ii), we have

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=\sin^2\theta\times\frac{-25\sin 5\theta\sin\theta-5\cos\theta\cos 5\theta}{\sin^3\theta}$$
$$-\cos\theta\left(-\frac{5\cos 5\theta}{\sin\theta}\right)$$

$$=\frac{-25\sin 5\theta\sin\theta-5\cos\theta\cos 5\theta+5\cos\theta\cos 5\theta}{\sin\theta}$$

$$=\frac{-25\sin 5\theta\sin\theta}{\sin\theta}=-25\sin 5\theta=-25y$$

Therefore, $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=-25y$

4. (b) : Let $z = x + iy$

$\therefore\quad z - 3i = x + iy - 3i = x + i(y - 3)$

Now, $\arg(x+i(y-3))=\frac{3\pi}{4}\ \Rightarrow \tan^{-1}\left(\frac{y-3}{x}\right)=\frac{3\pi}{4}$

$\Rightarrow\quad \frac{(y-3)}{x}=-1\ \Rightarrow\ x+y=3$ ...(i)

Again, $2z - 1 - 2i = 2(x + iy) - 1 - 2i = (2x - 1) + i(2y - 2)$

$\therefore\ \arg(2z-1-2i)=\frac{\pi}{4}\ \Rightarrow\ \arg\{(2x-1)+i(2y-2)\}=\frac{\pi}{4}$

$\Rightarrow\quad \tan^{-1}\left(\frac{2y-2}{2x-1}\right)=\frac{\pi}{4}\ \Rightarrow\ \frac{2y-2}{2x-1}=1$

$\Rightarrow\quad 2x - 2y = -1$ ...(ii)

Solving (i) and (ii), we get $x=\frac{5}{4},\ y=\frac{7}{4}$

5. (c) : Here, the given equation is

$f(x)+2f\left(\frac{1}{x}\right)=3x,\ x\neq 0$

Now replacing x to $1/x$ in the above equation, we get

$f\left(\frac{1}{x}\right)+2f(x)=\frac{3}{x}$

Eliminating $f\left(\frac{1}{x}\right)$ from the above two equations, we get

$3f(x)=\frac{6}{x}-3x\ \Rightarrow\ f(x)=\frac{2}{x}-x$

Now to get the elements of S, we solve $f(x) = f(-x)$

$\Rightarrow\quad \frac{2}{x}-x=-\frac{2}{x}+x\ \Rightarrow\ \frac{2}{x}-x=0$

$\Rightarrow\quad x^2=2\ \Rightarrow\ x=\pm\sqrt{2}$

6. (a) : The equation of the tangent at $P(a\cos\theta, b\sin\theta)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ ...(i)

The combined equation of the chords joining the points of intersection of (i) and the auxiliary circle $x^2 + y^2 = a^2$ is

$$x^2+y^2=a^2\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)^2.$$

The lines given by this equation are at right angle.

$\therefore$ Coefficient of x^2 + Coefficient of y^2 = 0

$\Rightarrow\quad 1-\cos^2\theta+1-a^2\left(\frac{\sin^2\theta}{b^2}\right)=0$

$\Rightarrow\quad \sin^2\theta+1-\frac{a^2}{b^2}\sin^2\theta=0\ \Rightarrow\ \sin^2\theta\left(1-\frac{a^2}{b^2}\right)+1=0$

$\Rightarrow\quad -\sin^2\theta(a^2-b^2)+b^2=0$

$\Rightarrow\quad -a^2e^2\sin^2\theta+a^2(1-e^2)=0$

$\Rightarrow\quad 1=e^2(1+\sin^2\theta)\ \Rightarrow\ e=(1+\sin^2\theta)^{-1/2}$

7. (d) : Here range = r = largest value – smallest value $=\text{Max}\,|x_i-x_j|$, where $(i\neq j)$

And $S^2=\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$

Now, $(x_i-\bar{x})^2=\left[x_i-\frac{x_1+x_2+\ldots+x_n}{n}\right]^2$

$=1/n^2[(x_i-x_1)+(x_i-x_2)+\ldots+(x_i-x_n)]^2$

$=1/n^2[(x_i-x_1)+(x_i-x_2)+\ldots+(x_i-x_{i-1})+(x_i-x_{i+1}) + \ldots + (x_i - x_n)]^2$

$\Rightarrow\quad (x_i-\bar{x})^2\leq 1/n^2[(n-1)r]^2$ $[\because\ |x_i - x_j| \leq r]$

MtG

ONLINE TEST SERIES

Practice Part Syllabus/ Full Syllabus
24 Mock Tests for

JEE Main

Now on your android Smart phones
with the same login of web portal.

Log on to test.pcmbtoday.com

$$\Rightarrow \quad \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 \le \frac{1}{n^2(n-1)}\sum[(n-1)r]^2$$

(Summing up and dividing by $(n-1)$ both sides)

$$=\frac{1}{n^2}\frac{1}{n-1}n(n-1)^2r^2$$

$$=\frac{n-1}{n}r^2<\frac{n}{n-1}r^2 \qquad \left(\because \forall n>1, n>\frac{1}{n}\right)$$

Therefore, $S^2<\frac{n}{n-1}\cdot r^2$ or $S<r\sqrt{\frac{n}{n-1}}$

8. (a) : Let $\vec{b}=x\hat{i}+y\hat{j}+z\hat{k}$

Then $\vec{a}\cdot\vec{b}=3 \Rightarrow x+y+z=3$, and

$\vec{a}\times\vec{b}=\vec{c} \Rightarrow (z-y)\hat{i}+(x-z)\hat{j}+(y-x)\hat{k}=\hat{j}-\hat{k}$

$\Rightarrow z-y=0, x-z=1$ and $y-x=-1$

Solving these equations with $x+y+z=3$, we get

$x=5/3, y=2/3$ and $z=2/3$

Therefore, $\vec{b}=\frac{1}{3}\left(5\hat{i}+2\hat{j}+2\hat{k}\right)$

9. (b) : Let the foot of the perpendicular be L.

The general point on the straight line

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9} \text{ is}$$

$$(\lambda-5, 4\lambda-3, -9\lambda+6) \qquad \text{...(i)}$$

Let $L=(\lambda-5, 4\lambda-3, -9\lambda+6)$

$\therefore$ The direction ratios of AL are

$\lambda-7, 4\lambda-7, -9\lambda+7$

Since AL is perpendicular to the line (i), therefore

$1(\lambda-7)+4(4\lambda-7)-9(-9\lambda+7)=0$

$\Rightarrow \lambda=1$

Thus, $L=(\lambda-5, 4\lambda-3, -9\lambda+6)=(-4, 1, -3)$

10. (c) : Let V be the volume of the parallelepiped formed by the given vectors. Let $\vec{\alpha}=\hat{i}+a\hat{j}+\hat{k}, \vec{\beta}=\hat{j}+a\hat{k}$ and $\vec{\gamma}=a\hat{i}+\hat{k}$.

$$\text{Then, } V=[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=1+a^3-a$$

Clearly, V will be minimum when $1+a^3-a$ is minimum.

Now, $V=1+a^3-a \Rightarrow \frac{dV}{da}=3a^2-1$ and $\frac{d^2V}{da^2}=6a$

For maximum or minimum value of V, we must have

$$\frac{dV}{da}=0 \Rightarrow a=\pm\frac{1}{\sqrt{3}}$$

$$\text{Clearly, } \left(\frac{d^2V}{da^2}\right)_{a=\frac{1}{\sqrt{3}}}=\frac{6}{\sqrt{3}}>0$$

Hence, V is minimum for $a=1/\sqrt{3}$.

11. (c) : Let E_1 be the event that the flight departs on time, E_2 be the event that the flight arrives on time.

$\therefore$ $P(E_1)=0.9, P(E_2)=0.8$ and $P(E_1\cap E_2)=0.7$

$$\text{Now, } P(E_2\,|\,E_1)=\frac{P(E_1\cap E_2)}{P(E_1)}=\frac{0.7}{0.9}=\frac{7}{9}$$

12. (a) : We have, $|A|=\begin{vmatrix}0 & 1 & -1\\ 2 & 1 & 3\\ 3 & 2 & 1\end{vmatrix}=0+7-1=6$

$\therefore$ $(A(\text{adj}A)A^{-1})A=(A(\text{adj}A))(A^{-1}A)$

$\Rightarrow$ $(A(\text{adj}A)A^{-1})A=(|A|I)I$

$$\Rightarrow (A(\text{adj}A)A^{-1})A=|A|I=\begin{bmatrix}6 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & 6\end{bmatrix}=2\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$$

13. (a): The given system of linear equations has a non-trivial solution. Therefore, $\begin{vmatrix}1 & 4a & a\\ 1 & 3b & b\\ 1 & 2c & c\end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix}0 & 4a-3b & a-b\\ 0 & 3b-2c & b-c\\ 1 & 2c & c\end{vmatrix}=0$$

[Applying $R_1\to R_1-R_2$ and $R_2\to R_2-R_3$]

$\Rightarrow$ $(4a-3b)(b-c)-(3b-2c)(a-b)=0$

$\Rightarrow$ $ab-2ac+bc=0$

$\Rightarrow$ $\frac{1}{a}+\frac{1}{c}=\frac{2}{b} \Rightarrow a,b,c$ are in H.P.

14. (a) : Given that

$$\frac{dy}{dx}=\frac{y}{x}+\sec\left(\frac{y}{x}\right), \; x>0 \qquad \text{...(i)}$$

Let $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

(i) becomes, $v+x\frac{dv}{dx}=v+\sec v$

$\Rightarrow \cos v\,dv=\frac{1}{x}dx$

Integrating both sides, we get

$$\sin v=\log x+c \Rightarrow \sin\left(\frac{y}{x}\right)=\log x+c \qquad \text{...(ii)}$$

It passes through the point $\left(1, \frac{\pi}{6}\right)$.

Therefore $\sin\frac{\pi}{6}=\log 1+c \Rightarrow c=\frac{1}{2}$

Now (ii) becomes, $\sin\left(\frac{y}{x}\right)=\log x+\frac{1}{2}$.

15. (b) : $\because h(x) = f(x) - \{f(x)\}^2 + \{f(x)\}^3$

$\therefore \quad h'(x) = f'(x) - 2f(x)f'(x) + 3\{f(x)\}^2f'(x)$

$$= 3f'(x)\left[\{f(x)\}^2 - 2f(x)\cdot\frac{1}{3} + \left(\frac{1}{3}\right)^2 + \frac{1}{3} - \frac{1}{9}\right]$$

$$= 3f'(x)\left[\left\{f(x) - \frac{1}{3}\right\}^2 + \frac{2}{9}\right]$$

We have given that $f'(x) > 0$

Thus $h'(x) > 0$ for all real values of x.

Therefore $h(x)$ is an increasing function for all real values of x.

16. (b) : We have, $f(x) = |x|^{|\tan x|}$

$\Rightarrow f(x) = (-x)^{-\tan x}$ [$\because$ In the neighbourhood of $x = -\frac{\pi}{6}$, $|x| = -x$ and $|\tan x| = -\tan x$]

Now, $\log f(x) = -\tan x \log(-x)$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{f(x)}\cdot f'(x) = \left[-\sec^2 x\cdot\log(-x) - \frac{\tan x}{x}\right]$$

$$\Rightarrow \quad f'(x) = (-x)^{-\tan x}\left[-\sec^2 x\cdot\log(-x) - \frac{\tan x}{x}\right]$$

$$\Rightarrow \quad f'\left(-\frac{\pi}{6}\right) = \left(\frac{\pi}{6}\right)^{\frac{1}{\sqrt{3}}}\left\{-\frac{4}{3}\log\frac{\pi}{6} - \frac{2\sqrt{3}}{\pi}\right\}$$

17. (a) : Consider

$(1 - x^3)^n = C_0 - C_1x^3 + C_2x^6 - C_3x^9 + \ldots$

$\Rightarrow \quad x^2(1 - x^3)^n = C_0x^2 - C_1x^5 + C_2x^8 - C_3x^{11} + \ldots$

Now integrating both sides w.r.t. x from limits 0 to 2, we get

$$\int_0^2 x^2(1-x^3)^n\,dx = \left[C_0\cdot\frac{x^3}{3} - C_1\cdot\frac{x^6}{6} + C_2\cdot\frac{x^9}{9} - \ldots\right]_0^2$$

$$= \left(C_0\cdot\frac{2^3}{3} - C_1\cdot\frac{2^6}{6} + C_2\cdot\frac{2^9}{9} - \ldots\right) \qquad \ldots\text{(i)}$$

Now, in L.H.S. put $1 - x^3 = z \Rightarrow \quad x^2dx = -\frac{1}{3}dz$

$$\therefore \quad \text{L.H.S} = \int_1^{-7} z^n\left(-\frac{1}{3}\right)dz = \frac{1}{3}\int_{-7}^{1} z^n dz = \frac{1}{3}\cdot\frac{z^{n+1}}{n+1}\Bigg|_{-7}^{1}$$

$$= \frac{1}{3(n+1)} - \frac{1}{3}\cdot\frac{(-7)^{n+1}}{n+1} = \frac{1}{3(n+1)}(1-(-7)^{n+1})$$

Now (i) becomes $\frac{1}{3(n+1)}(1-(-7)^{n+1})$

$$= C_0\cdot\frac{2^3}{3} - C_1\cdot\frac{2^6}{6} + C_2\cdot\frac{2^9}{9} - \ldots$$

Multiplying L.H.S and R.H.S by $\frac{3}{2^3}$, we get

$$C_0 - C_1\frac{2^3}{2} + C_2\frac{2^6}{3} - \ldots(n+1)\text{ terms} = \frac{1-(-7)^{n+1}}{8(n+1)}$$

18. (c) : Let the line $3x + y + 5 = 0$ cuts the circle $x^2 + y^2 = 16$ at the points A and B, hence, $\overline{AB}$ is a chord of the circle. The mid-point of the chord $\overline{AB}$ is $\left(-\frac{3}{2}, -\frac{1}{2}\right)$(the foot of the perpendicular from

PUBLIC NOTICE JEE MAIN 2022

Sub: Rescheduling the Dates for Joint Entrance Examination (Main) – 2022 Session 1 and Session 2

In continuation to the Public Notice dated: 14 March 2022, National Testing Agency has decided to reschedule the dates of JEE (Main) – 2022 Session 1 and Session 2 based on the numerous representations received from the candidates. The revised schedule is as follows:

Exam/Session	Earlier Dates	Revised Dates	Intimation of City	Downloading of Admit Card
JEE (Main) – 2022 Session 1	21, 24, 25, 29 April, and 1, 4 May 2022	20, 21, 22, 23, 24, 25, 26, 27, 28, and 29 June 2022	To be intimated later on the JEE (Main) website.	
JEE (Main) – 2022 Session 2	24, 25, 26, 27, 28, and 29 May 2022	21, 22, 23, 24, 25, 26, 27, 28, 29 and 30 July 2022		

The Candidates are advised to keep visiting the official websites of NTA (www.nta.ac.in) and (https://jeemain.nta.nic.in/) for the latest updates.

For further clarification related to JEE (Main) – 2022, the candidates can also contact 011-40759000/011-69227700 or email at jeemain@nta.ac.in.

the centre to the chord) and length of the chord

$$\overline{AB}=2\times\sqrt{16-\frac{10}{4}}=2\sqrt{\frac{27}{2}}.$$

$\therefore$ Centre of the circle, whose diameter is AB, is $\left(-\frac{3}{2},-\frac{1}{2}\right)$ and its radius $=\frac{1}{2}AB=\sqrt{\frac{27}{2}}.$

Thus the equation of the circle is

$$\left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{27}{2}\Rightarrow x^2+y^2+3x+y=11.$$

19. (a) : We have, $\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$

$$\Rightarrow \frac{dp(t)}{dt}=\frac{p(t)-400}{2}\quad\Rightarrow \int\frac{dp(t)}{p(t)-400}=\frac{1}{2}\int dt$$

$$\Rightarrow \log\left|p(t)-400\right|=\frac{1}{2}t+\log c$$

$$\Rightarrow \log\left|\frac{p(t)-400}{c}\right|=\frac{1}{2}t\quad\Rightarrow\quad p(t)=ce^{t/2}+400$$

When $p(0)=100$, we get,

$100=ce^{0/2}+400\quad\Rightarrow\ c=-300$

$\therefore$ The required solutions is $p(t)=400-300e^{t/2}$

20. (c) : The equation of the plane passing through the point (0, 1, 0) is

$ax+b(y-1)+cz=0$ ...(i)

Since (i) is also passing through the point(3, 4, 1)

$\therefore\ 3a+3b+c=0$ ...(ii)

Again the plane (i) is parallel to the line

$$\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$$

$\therefore\ 2a+7b+5c=0$ ...(iii)

Now solving (ii) and (iii), we get

$$\frac{a}{15-7}=\frac{b}{2-15}=\frac{c}{21-6}\quad\Rightarrow\quad\frac{a}{8}=\frac{b}{-13}=\frac{c}{15}=\lambda$$

$\therefore\ a=8\lambda,\ b=-13\lambda,\ c=15\lambda$

Now (i) becomes

$8\lambda x-13\lambda y+15\lambda z=-13\lambda$

$\Rightarrow\ 8x-13y+15z+13=0$

21. (0) : We have,

$$\sin\left\{\sin^{-1}(\log_{1/2}x)\right\}+2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2}-\frac{3}{2}\right)\right\}\right|=0$$

The two terms on L.H.S. of the given equation are meaningful, if $-1\le\log_{1/2}x\le1$ and $-1\le\frac{x}{2}-\frac{3}{2}\le1$

$$\Rightarrow 2\ge x\ge\frac{1}{2}\text{ and }1\le x\le5\ \Rightarrow\ 1\le x\le2$$

Now, $1\le x\le2$

$$\Rightarrow \frac{1}{2}\le\frac{x}{2}\le1\ \Rightarrow\ -\frac{1}{2}\le\frac{x}{2}-1\le0$$

$$\Rightarrow -\frac{\pi}{6}\le\sin^{-1}\left(\frac{x}{2}-1\right)\le0$$

$$\Rightarrow \frac{\sqrt{3}}{2}\le\cos\left\{\sin^{-1}\left(\frac{x}{2}-1\right)\right\}\le1$$

$$\Rightarrow \sqrt{3}\le2\cos\left\{\sin^{-1}\left(\frac{x}{2}-1\right)\right\}\le2$$

$$\Rightarrow \sqrt{3}\le2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2}-1\right)\right\}\right|\le2\qquad\text{...(i)}$$

Again, $1\le x\le2$

$$\Rightarrow\ -1\le\log_{1/2}x\le0\ \Rightarrow\ -\frac{\pi}{2}\le\sin^{-1}(\log_{1/2}x)\le0$$

$$\Rightarrow\ -1\le\sin\{\sin^{-1}(\log_{1/2}x)\}\le0\qquad\text{...(ii)}$$

Adding (i) and (ii), we get

$$\sqrt{3}-1\le\sin\{\sin^{-1}(\log_{1/2}x)\}+2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2}-1\right)\right\}\right|\le2$$

$$\Rightarrow\ \sin\{\sin^{-1}(\log_{1/2}x)\}+2\left|\cos\left\{\sin^{-1}\left(\frac{x}{2}-\frac{3}{2}\right)\right\}\right|\ne0$$

for any x. Hence the given equation has no solution.

22. (4) : It is given that a, b, c are in G.P., and the A.M. of a, b, c is $b+2$.

$$\therefore\ b^2=ac\text{ and }\frac{a+b+c}{3}=b+2$$

$\Rightarrow\ b^2=ac$ and $a+c=2b+6$

$$\Rightarrow\ c=\frac{b^2}{a}\text{ and }a-2b+c=6$$

$$\Rightarrow\ a-2b+\frac{b^2}{a}=6\ \Rightarrow\ \left(\frac{b}{a}\right)^2-2\left(\frac{b}{a}\right)+1=\frac{6}{a}$$

$$\Rightarrow\ \left(\frac{b}{a}-1\right)^2=\frac{6}{a}$$

$$\Rightarrow\ \frac{6}{a}\text{ is a perfect square of integer }\left[\because\ \frac{b}{a}\text{ is an integer}\right]$$

$$\Rightarrow\ a=6\ \ \therefore\ \frac{a^2+a-14}{a+1}=\frac{36+6-14}{6+1}=4$$

23. (1) : We know that

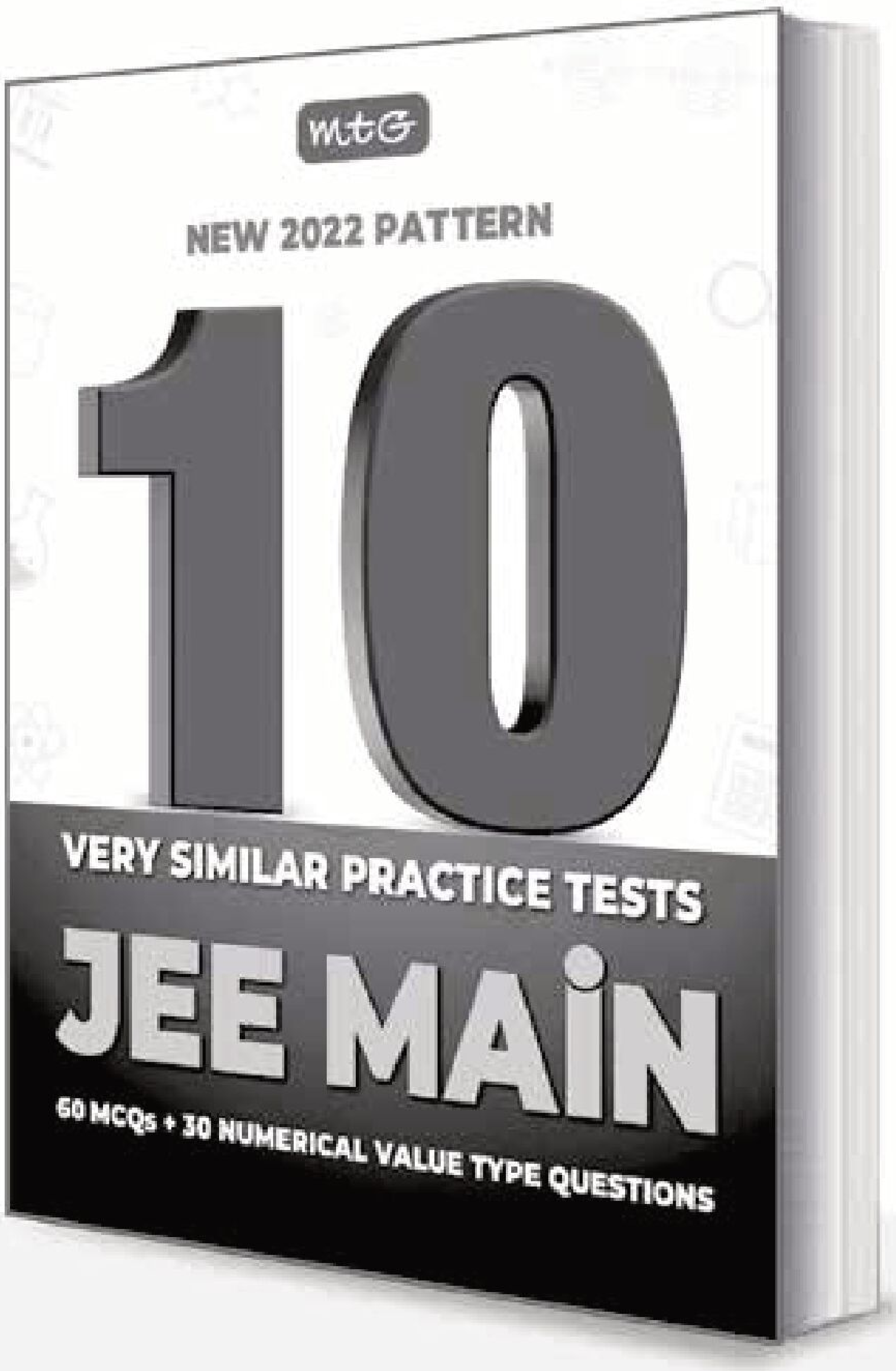
$$\tan3A=\frac{3\tan A-\tan^3A}{1-3\tan^2A}$$

mtG

10 Very Similar
Practice Tests
JEE Main

Visit
www.mtg.in
for latest offers
and to buy
online!

₹ 200/-


mtG
NEW 2022 PATTERN
10
VERY SIMILAR PRACTICE TESTS
JEE MAIN
60 MCQs + 30 NUMERICAL VALUE TYPE QUESTIONS

HIGHLIGHTS

- 10 Very Similar Practice Tests as per the latest pattern of NTA JEE Main (60 MCQs+ 30 Numerical Value Type Questions)
- OMR sheet provided at the end of each test
- Detailed solutions of each practice test included

Available at all leading book shops throughout India. To buy online visit www.mtg.in.
For more information or for help in placing your order,
call 0124-6601200, 1800-10-38673 (Toll Free) or e-mail *info@mtg.in*

Also available on
Flipkart, Snapdeal, Amazon,
Infibeam on cash on delivery

$$\Rightarrow \tan 30^\circ = \frac{3\tan 10^\circ - \tan^3 10^\circ}{1 - 3\tan^2 10^\circ} \qquad (\text{Putting } A = 10^\circ)$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3\tan 10^\circ - \tan^3 10^\circ}{1 - 3\tan^2 10^\circ}$$

$$\Rightarrow 1 - 3\tan^2 10^\circ = \sqrt{3}(3\tan 10^\circ - \tan^3 10^\circ)$$

$$\Rightarrow 1 + 9\tan^4 10^\circ - 6\tan^2 10^\circ = 3(9\tan^2 10^\circ + \tan^6 10^\circ - 6\tan^4 10^\circ)$$

$$\Rightarrow 3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ = 1$$

24. (3) : Here, $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}\,dx$

$$= \int \frac{1}{\sqrt{3^2 - (\sin x + \cos x)^2}}\,d(\cos x + \sin x)$$

$$= \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

25. (2) : We know that,

$$x^2 - x - 2 = \begin{cases} (x+1)(x-2) < 0, & \text{if } x \in (-1, 2) \\ (x+1)(x-2) > 0, & \text{if } x \in (2, \infty) \end{cases}$$

Thus the function can be redefined as

$$f(x) = \begin{cases} a \text{ if } x < 2 \\ b \text{ if } x = 2 \\ 1 \text{ if } x > 2 \end{cases}$$

(Since in the neighbourhood of 2 on left side $|x^2 - x - 2| = -(x^2 - x - 2)$ and $x - [x] = x - 2$ if $x \in (2, 3)$)

∴ $f(x)$ is continuous at $x = 2 \Rightarrow a = b = 1$

26. (26) : Any tangent to

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \text{ is } y = mx + \sqrt{9m^2 + 4} \qquad \text{...(i)}$$

It touches $\frac{x^2}{4} + \frac{y^2}{9} = 1$. So, (i) is of the form

$$x = m'y + \sqrt{9m'^2 + 4} \qquad \text{...(ii)}$$

Comparing (i) and (ii), $\frac{m}{-1} = \frac{-1}{m'} = \frac{\sqrt{9m^2 + 4}}{\sqrt{9m'^2 + 4}}$

$$\Rightarrow mm' = 1 \text{ and } m^2 = \frac{9m^2 + 4}{9m'^2 + 4}$$

$$\Rightarrow m^2(9m'^2 + 4) = 9m^2 + 4$$

$$\Rightarrow 9 + 4m^2 = 9m^2 + 4 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1 \qquad (\because mm' = 1)$$

Therefore the common tangents are

$$y = x \pm \sqrt{13} \text{ and } y = -x \pm \sqrt{13}$$

Clearly the above 4 lines form a square of sides $\sqrt{26}$ units *i.e.*, of area 26 square units.

27. (12) : Given that $f(x)$ is continuous in $(-\infty, \infty)$ and $f'(x)$ exists in $(-\infty, \infty)$.

$$\therefore \quad f'(c) = \frac{f(6) - f(3)}{6 - 3}, \text{ where } c \in (3, 6)$$

$$\Rightarrow \frac{f(6) - f(3)}{6 - 3} \ge 6 \Rightarrow f(6) - f(3) \ge 18 \Rightarrow f(6) \ge 12$$

$[\because f(3) = -6 \text{ and } f'(x) \ge 6 \Rightarrow f'(c) \ge 6]$

28. (900) : The word ARRANGE is consist of 7 letters. Among these 7 letters there are two Rs. and two As. Considering the two letters (RR) as a single letter, the given word can be arranged in $\frac{6!}{2!}$ ways, where two R are always together.

∴ The number of words where two R do not come together are $\frac{7!}{2!2!} - \frac{6!}{2!} = \frac{6!}{2!}\left(\frac{7}{2} - 1\right) = \frac{6!}{2!} \cdot \frac{5}{2} = 900.$

29. (0) : Let $I = \int_0^{\pi} e^{\sin^2 x} \cos^3 x\,dx$

$$= \int_0^{\pi/2} [e^{\sin^2 x} \cos^3 x + e^{\sin^2 (\pi - x)} \cos^3(\pi - x)]dx$$

$$\left[\because \int_0^{2a} f(x)\,dx = \int_0^{2a} f(2a - x)\,dx\right]$$

$$= \int_0^{\pi/2} [e^{\sin^2 x} \cos^3 x - e^{\sin^2 x} \cos^3 x]dx = 0$$

30. (1.10) : Here, we have

$$C = k\tan\theta \Rightarrow \frac{dC}{d\theta} = k\sec^2\theta \Rightarrow \frac{dC}{d\theta}\delta\theta = k\sec^2\theta\,\delta\theta$$

$$\Rightarrow \delta C = k\sec^2\theta\,\delta\theta \qquad \left[\because \delta C = \frac{dC}{d\theta}\delta\theta\right]$$

$$\text{Now, } \frac{\delta C}{C} \times 100 = \frac{k\sec^2\theta\,\delta\theta}{k\tan\theta} \times 100 = \frac{\sec^2\theta}{\tan\theta}\delta\theta \times 100$$

$$= 2 \times \frac{\pi}{4}\frac{\delta\theta}{\theta} \times 100 = \frac{\pi}{2} \times 0.7 = 1.57 \times 0.7 = 1.099$$

∴ Percentage of error in C is 1.1.

Exam Dates
Session-1
20th to 29th June
Session-2
21st to 30th July

JEE Main 2022

MOCK TEST PAPER

SECTION - A

MULTIPLE CHOICE QUESTIONS

1. If $\int_0^{\pi/4}\sqrt{\tan x}\,dx = a\pi + b\ln\left(\sqrt{2}+1\right)$, then $a + b =$

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}}$ (d) $\frac{-1}{2\sqrt{2}}$

2. The general solution of the differential equation $(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$ is $2xe^{f(y)}=e^{2f(y)}+c$, then the area of the region bounded by the curves $x=f(y), y=\pm\sqrt{3}$ and y-axis is

(a) $\frac{\pi}{\sqrt{3}}-\log 2$ (b) $\frac{2\pi}{\sqrt{3}}-\log 4$

(c) $\frac{\pi}{\sqrt{3}}+\log 2$ (d) $\frac{2\pi}{\sqrt{3}}$

3. The eccentricity of the ellipse whose axes are the coordinate axes and which passes through the points (2, 2) and (3, 1) is

(a) $\frac{1}{3}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\frac{2}{5}$ (d) $\sqrt{\frac{2}{5}}$

4. Let a, b, c be real numbers with $a^2 + b^2 + c^2 = 1$ and $\begin{vmatrix} ax-by-c & bx+ay & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix}=0$

then $ax + by + c =$

(a) 1 (b) –1 (c) 0 (d) 2

5. The area of the triangle formed by the line $x + y = 3$ and the angle bisectors of the pair of lines $x^2 - y^2 + 2y = 1$ is

(a) 2 (b) 4 (c) 6 (d) 8

6. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$ does not exceed

(a) 4 (b) 9 (c) 8 (d) 6

7. $\lim\limits_{x\to 0}\frac{5\sin x-7\sin 2x+3\sin 3x}{x^2\sin x}=$

(a) 2 (b) –3 (c) 4 (d) –5

8. Let a, b, c be positive numbers. If $ax^2+\frac{b}{x}\geq c$ for all $x > 0$, then the minimum value of $\frac{ab^2}{c^3}$ is

(a) 1 (b) $\frac{2}{9}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

9. If $\frac{bc}{ad}=\frac{b+c}{a+d}=3\left(\frac{b-c}{a-d}\right)$, then a, b, c, d are in

(a) A.P. (b) G.P. (c) H.P. (d) A.G.P

10. The least value of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ is

(a) $\frac{\pi^2}{64}$ (b) $\frac{\pi^3}{16}$ (c) $\frac{\pi^3}{8}$ (d) $\frac{\pi^3}{32}$

11. A variable plane makes intercepts on x, y and z axes and it makes a tetrahedron of volume 64 cu. units. The locus of foot of perpendicular from origin on this plane is

(a) $(x^2 + y^2 + z^2)^3 = 384\,xyz$

(b) $xyz = 681$

(c) $(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)^2=16$

(d) $xyz\,(x + y + z) = 81$

12. Four students of class IV, 5 students of class V and 6 students of class VI sit in a row. The number of ways they can sit in a row so that the students belonging to same class are together is

(a) 4! 5! 6!
(b) $\dfrac{15!}{4!5!6!}$
(c) $\dfrac{15!}{3!4!5!6!}$
(d) 3! 4! 5! 6!

13. Let A be a 3×3 matrix given by $A = (a_{ij})_{3\times3}$. If for every column vector X satisfies $X' AX = O$ and $a_{12} = 2008$, $a_{13} = 2010$ and $a_{23} = -2012$, then the value of $a_{21} + a_{31} + a_{32}$ is
(a) -6
(b) 2006
(c) -2006
(d) 0

14. The number of roots of the equation
$$\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2} \text{ in } [0, 2\pi] \text{ is}$$
(a) 2
(b) 3
(c) 4
(d) 5

15. If $g(x) = \left(4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7\right)^{1/7}$, then the value of $g(g(100))$ is equal to
(a) -1
(b) 0
(c) 1
(d) 100

16. Let X denote the number of times tail appear in n tosses of a fair coin. If $P(X = 1)$, $P(X = 2)$ and $P(X = 3)$ are in A.P., then the value of n is
(a) 9
(b) 2
(c) 7
(d) None of these

17. If $f(x) = \cos(\ln x)$, then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right] =$
(a) -1
(b) 1/2
(c) -2
(d) 0

18. Let $x, y \in R$. If $\dfrac{(1+i)x - 2i}{3+i} + \dfrac{(2-3i)y + i}{3-i} = i$, then $x + y =$
(a) 1
(b) 2
(c) -1
(d) -3

19. Let $f(x) = x^3 - x^2 + x + 1$ and
$$g(x) = \begin{cases} \max\{f(t) : 0 \le t \le x\}, & 0 \le x \le 1 \\ 3 - x, & 1 < x \le 2 \end{cases}$$
Then in $[0, 2]$, $g(x)$ is
(a) not bounded
(b) bounded, but not continuous
(c) continuous, but not differentiable
(d) differentiable

20. In a triangle ABC, if $r_1 = 3$ and $s = 4$, then $\cos A =$
(a) 4/5
(b) 3/5
(c) 24/25
(d) 7/25

SECTION - B

NUMERICAL TYPE

Attempt any 5 questions out of 10.

21. The area enclosed by the curve $xy^2 = 4(2 - x)$ and y-axis is λ sq. unit, then the value of $\cos(1824\lambda)$ must be

22. If $\dfrac{dy}{dx} = xy + x^3y^3$ and $y(0) = 1$, then $3\left(y\left(\frac{1}{2}\right)\right)^2$ is

23. The tangents drawn from a point P to the ellipse $9x^2 + 16y^2 = 144$ make angles θ_1, θ_2 with the major axis. If $\theta_1 + \theta_2 = 2\alpha$, then the locus of P is $x^2 - 2xy \cot 2\alpha - y^2 = \lambda$, where λ is

24. Let $g : (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ defined as $g(x) = 2\tan^{-1}(e^x) - \frac{\pi}{2}$ and f is inverse of g then the value of $f'(0)$ is

25. The last digit of the coefficient of x^4 in the expansion of $(1 + 2x + 3x^2)^{10}$ is

26. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines
$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$
and $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-5}{5}$ is $\sqrt{6}$, then $|d|$ is

27. If the quadratic equations $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$, $(b \ne c)$ have a common root, then $(a + 4b + 4c)$ is

28. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$, and $(A + B)^2 = A^2 + B^2$, find $a + b$.

29. Mother, father and son line up at random for a family picture
E : son on one end, F : father in middle. $P(E/F)$ is

30. Let α be the smallest positive integer x such that $\tan(19x)° = \dfrac{\cos 96° + \sin 96°}{\cos 96° - \sin 96°}$. The last digit of α is

SOLUTIONS

1. (d) : $\displaystyle\int_0^{\pi/4} \sqrt{\tan x}\, dx = \int_0^1 \frac{2t^2}{t^4 + 1} dt$, $\tan x = t^2$

$$=\int_0^1 \frac{t^2+1+t^2-1}{t^4+1}dt = \int_0^1 \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt + \int_0^1 \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt$$

$$=\int_0^1 \frac{d\left(t-\frac{1}{t}\right)}{\left(t-\frac{1}{t}\right)^2+2} + \int_0^1 \frac{d\left(t+\frac{1}{t}\right)}{\left(t+\frac{1}{t}\right)^2-2}$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)\Bigg|_0^1 + \frac{1}{2\sqrt{2}}\ln\left(\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right)\Bigg|_0^1$$

$$=\frac{\pi}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)=\frac{\pi}{2\sqrt{2}}-\frac{1}{\sqrt{2}}\ln\left(\sqrt{2}+1\right)$$

$$\Rightarrow\ a+b=\frac{1}{2\sqrt{2}}-\frac{1}{\sqrt{2}}=\frac{-1}{2\sqrt{2}}.$$

2. (b) : $\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$

$$\text{I.F.}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

$\therefore$ General solution is $x\cdot e^{\tan^{-1}y}=\int\frac{\left(e^{\tan^{-1}y}\right)^2}{1+y^2}dy$

$$\Rightarrow\ x.e^{\tan^{-1}y}=\frac{e^{2\tan^{-1}y}}{2}+c$$

$$\Rightarrow\ 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+c\quad\therefore\ f(y)=\tan^{-1}y$$

$$\therefore\ \text{Area}=\int_{-\sqrt{3}}^{\sqrt{3}}|\tan^{-1}y|\,dy=2\int_0^{\sqrt{3}}\tan^{-1}y\,dy$$

$$=2\left[(y\tan^{-1}y)_0^{\sqrt{3}}-\int_0^{\sqrt{3}}\frac{y}{1+y^2}dy\right]$$

$$=2\frac{\pi}{\sqrt{3}}-\left[\log(1+y^2)\right]_0^{\sqrt{3}}=\frac{2\pi}{\sqrt{3}}-\log 4$$

3. (d) : The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. It passes through (2, 2) and (3, 1).

$$\frac{4}{a^2}+\frac{4}{b^2}=1,\ \frac{9}{a^2}+\frac{1}{b^2}=1,\ \frac{1}{b^2}=1-\frac{9}{a^2}$$

$$\therefore\ \frac{4}{a^2}+4\left(1-\frac{9}{a^2}\right)=1\ \Rightarrow\ \frac{1}{a^2}=\frac{3}{32},\ \frac{1}{b^2}=\frac{5}{32}$$

Since, $b^2=a^2(1-e^2)\Rightarrow 32/5=32\,(1-e^2)/3$

$\Rightarrow\ 1-e^2=3/5\ \Rightarrow\ e^2=2/5\Rightarrow e=\sqrt{2/5}.$

4. (c) : Applying $R_1\to aR_1+bR_2+cR_3$, gives

$$\begin{vmatrix} x & y & 1\\ bx+ay & -ax+by-c & cy+b\\ cx+a & cy+b & -ax-by+c\end{vmatrix}=0$$

Applying $R_2\to R_2-bR_1,\ R_3\to R_3-cR_1$, gives

$$\begin{vmatrix} x & y & 1\\ ay & -ax-c & cy\\ a & b & -ax-by\end{vmatrix}=0$$

Applying $C_3\to C_3+xC_1+yC_2$, gives

$$\begin{vmatrix} x & y & x^2+y^2+1\\ ay & -ax-c & 0\\ a & b & 0\end{vmatrix}=0$$

$$\therefore\ (x^2+y^2+1)\begin{vmatrix} ay & -ax-c\\ a & b\end{vmatrix}=0$$

$$\Rightarrow\ \begin{vmatrix} y & -ax-c\\ 1 & b\end{vmatrix}=0\ \text{(Since } x^2+y^2+1\neq 0)$$

$\Rightarrow\ ax+by+c=0.$

5. (a) : $x^2-y^2+2y=1$

$\Rightarrow x^2=y^2-2y+1=(y-1)^2$

$\therefore$ The pair is $x-y+1=0$, $x+y-1=0$

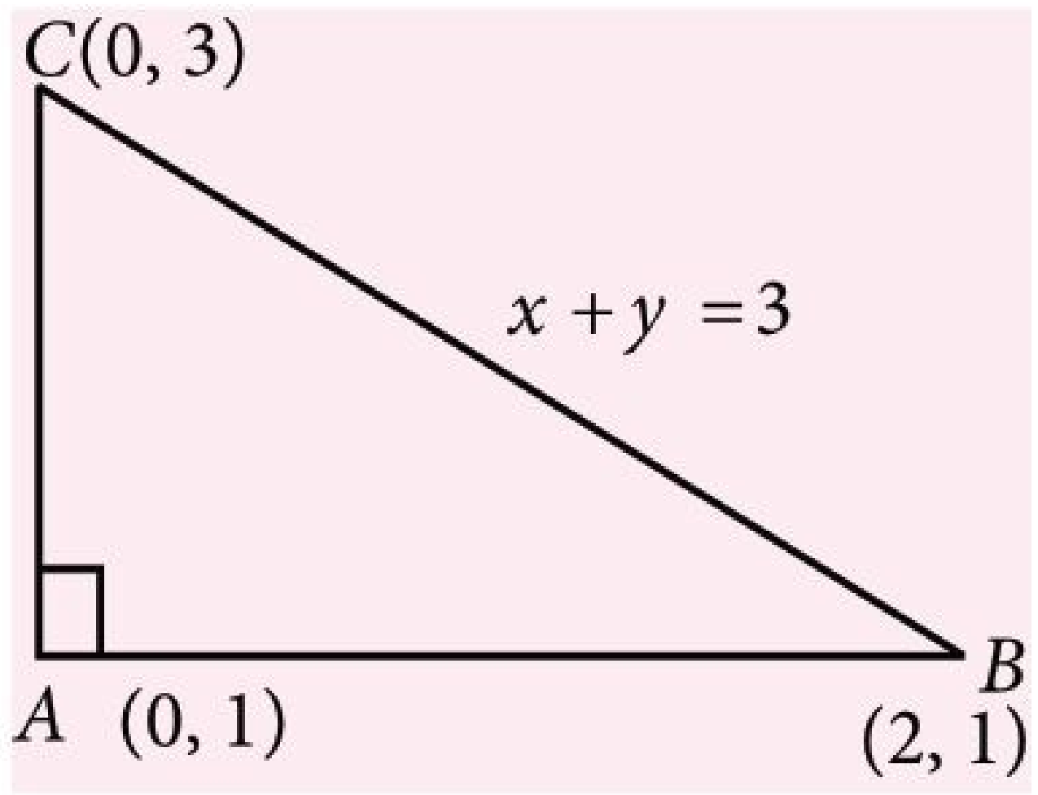
The angle bisectors between the pair are given by

$$\frac{x-y+1}{\sqrt{2}}=\pm\frac{(x+y-1)}{\sqrt{2}}$$

$\Rightarrow\ x=0$ and $y=1$.

The lines $x+y=3$, $x=0$ and $y=1$ form the triangle in the figure.

Area = (1/2) × 2 × 2 = 2.



6. (b) : $|\vec{a}+\vec{b}+\vec{c}|\geq 0$

$\Rightarrow\ |\vec{a}+\vec{b}+\vec{c}|^2=1+1+1+2\,(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq 0$

$\therefore\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\geq -3/2$ ...(i)

$|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2$

$=2|\vec{a}|^2+2|\vec{b}|^2+2|\vec{c}|^2-2\,(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

$=6-2\,(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\leq 6+3=9$, [using (i)]

7. (d) : $\lim\limits_{x\to 0}\frac{x}{\sin x}\,\frac{5\sin x-7\sin 2x+3\sin 3x}{x^3}$

$$=\lim_{x\to 0}\frac{5\sin x-7\sin 2x+3\sin 3x}{x^3}$$

$$=\lim_{x\to 0}\frac{5\cos x-14\cos 2x+9\cos 3x}{3x^2}\quad\text{(By L.H. Rule)}$$

$$=\lim_{x\to 0}\frac{-5\sin x+28\sin 2x-27\sin 3x}{6x}\quad\text{(By L.H. Rule)}$$

$$=\lim_{x\to 0}\frac{-5\cos x+56\cos 2x-81\cos 3x}{6}\quad\text{(By L.H. Rule)}$$

$=(-5+56-81)/6=-5.$

8. (c) : $f(x)=ax^2+\frac{b}{x}\Rightarrow f'(x)=2ax-\frac{b}{x^2}$

So, $f'(x)=0\Rightarrow 2ax=\frac{b}{x^2}\Rightarrow x=\left(\frac{b}{2a}\right)^{1/3}$

Now, $f''(x)=2a+\frac{2b}{x^3}$

$\Rightarrow\left[f''(x)\right]_{x=\left(\frac{b}{2a}\right)^{1/3}}=2a+4a=6a>0$

So, $f(x)$ is minimum at $x=\left(\frac{b}{2a}\right)^{1/3}$

Now, $\min f=a\left(\frac{b}{2a}\right)^{2/3}+b\left(\frac{2a}{b}\right)^{1/3}=\left(\frac{27ab^2}{4}\right)^{1/3}\ge c$

$\Rightarrow\frac{ab^2}{c^3}\ge\frac{4}{27}$

9. (c) : $\frac{bc}{ad}=\frac{b+c}{a+d}\Rightarrow\frac{b+c}{bc}=\frac{a+d}{ad}$

$\Rightarrow\frac{1}{b}+\frac{1}{c}=\frac{1}{a}+\frac{1}{d}$ ...(i)

$\frac{bc}{ad}=3\left(\frac{b-c}{a-d}\right)\Rightarrow\frac{a-d}{ad}=\frac{3(b-c)}{bc}$

$\Rightarrow\frac{1}{d}=\frac{1}{a}+3\left(\frac{1}{c}-\frac{1}{b}\right)$ ...(ii)

Let $\frac{1}{b}-\frac{1}{a}=\alpha$. Then (i) $\Rightarrow\frac{1}{d}-\frac{1}{c}=\alpha$

Substituting for $\frac{1}{b}$ and $\frac{1}{d}$ in (ii), we get

$\frac{1}{c}+\alpha=\frac{1}{a}+\frac{3}{c}-3\left(\frac{1}{a}+\alpha\right)\Rightarrow\frac{1}{c}=\frac{1}{a}+2\alpha$

$\therefore\ \frac{1}{b}=\frac{1}{a}+\alpha,\ \frac{1}{c}=\frac{1}{a}+2\alpha,\ \frac{1}{d}=\frac{1}{a}+3\alpha$

$\frac{1}{a},\frac{1}{b},\frac{1}{c},\frac{1}{d}$ are in A.P. and a, b, c, d are in H.P.

10. (d) : $(\sin^{-1}x)^3+(\cos^{-1}x)^3$

$=(\sin^{-1}x+\cos^{-1}x)((\sin^{-1}x)^2-\sin^{-1}x\cos^{-1}x+(\cos^{-1}x)^2)$

$$=\frac{\pi}{2}\left[\frac{\pi^2}{4}-3\sin^{-1}x\cos^{-1}x\right]$$

$$=\frac{\pi}{2}\left[\frac{\pi^2}{4}-3\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)\right]$$

$$=\frac{3\pi}{2}\left[\frac{\pi^2}{12}-\frac{\pi}{2}\sin^{-1}x+(\sin^{-1}x)^2\right]$$

$$=\frac{3\pi}{2}\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{12}-\frac{\pi^2}{16}\right]$$

$$=\frac{3\pi}{2}\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{48}\right]=f(x)$$

$m=f_{\min}=f\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi^3}{32}$

11. (a) : The equation of plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$

Volume of tetrahedron $OABC$ is
$v=abc/6=64\Rightarrow abc=384$
Foot of perpendicular from (0, 0, 0) on this plane is

$$\frac{x}{1/a}=\frac{y}{1/b}=\frac{z}{1/c}=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}=k$$

$\Rightarrow\ x=\frac{k}{a},y=\frac{k}{b},z=\frac{k}{c}$ and $\frac{1}{k}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$

$\Rightarrow\ \frac{1}{k}=\frac{x^2+y^2+z^2}{k^2}\Rightarrow x^2+y^2+z^2=k$

$\therefore\ (x^2+y^2+z^2)^3=k^3=abc\,xyz=384\,xyz$ is the required locus.

12. (d) : Class IV students can sit in 4! ways. Class V students in 5! ways and Class VI students in 6! ways. The classes can be permuted.
The desired number is 3! 4! 5! 6!.

13. (c) : Let $X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$. Then

$$(x_1\ x_2\ x_3)\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\Rightarrow\ a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + (a_{12} + a_{21})\, x_1x_2 + (a_{13} + a_{31})x_1x_3 + (a_{23} + a_{32})x_2x_3 = 0$$

It is true for every x_1, x_2, x_3.

$\therefore\ a_{11} = a_{22} = a_{33} = 0, a_{12} + a_{21} = 0, a_{31} + a_{13} = 0, a_{23} + a_{32} = 0$

$\Rightarrow$ A is a skew symmetric matrix.

$a_{21} = -2008, a_{31} = -2010, a_{32} = 2012$

$\therefore\ a_{21} + a_{31} + a_{32} = -2006$

14. (d) : $\cos 3x + \cos 2x = \sin(3x/2) + \sin(x/2)$

$\Rightarrow 2\cos(5x/2)\cos x/2 = 2\sin x\cos(x/2)$

$$\Rightarrow \cos\frac{x}{2}\left(\cos\frac{5x}{2} - \sin x\right) = 0$$

$\Rightarrow\ \cos(x/2) = 0 \Rightarrow x/2 = (2n+1)\pi/2$

$\Rightarrow\ x = (2n+1)\pi \Rightarrow x = \pi$ in $[0, 2\pi]$

And, $\cos\dfrac{5x}{2} = \sin x = \cos\left(\dfrac{\pi}{2} - x\right)$

$$\Rightarrow \frac{5x}{2} = 2n\pi \pm \left(\frac{\pi}{2} - x\right)$$

$$\Rightarrow \frac{5x}{2} = 2n\pi + \frac{\pi}{2} - x \Rightarrow \frac{7x}{2} = (4n+1)\frac{\pi}{2}$$

$$\Rightarrow\ x = (4n+1)\frac{\pi}{7} \Rightarrow x = \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}$$

or $\dfrac{5x}{2} = 2n\pi - \dfrac{\pi}{2} + x \Rightarrow \dfrac{3x}{2} = (4n-1)\dfrac{\pi}{2}$

$\Rightarrow x = (4n-1)\,\pi/3 \Rightarrow x = \pi$

$\therefore$ The roots are $\dfrac{\pi}{7}, \dfrac{5\pi}{7}, \pi, \dfrac{9\pi}{7}, \dfrac{13\pi}{7}$.

15. (d) : We have, $4\cos^4 x - 2\cos 2x - \cos 4x/2 - x^7$

$= 4\cos^4 x - 2(2\cos^2 x - 1) - (2\cos^2 2x - 1)/2 - x^7$

$= 4\cos^4 x - 4\cos^2 x + 2 - (2\cos^2 x - 1)^2 + (1/2) - x^7$

$= ((3/2) - x^7)$

$$\Rightarrow\ g(x) = \left(\frac{3}{2} - x^7\right)^{1/7}$$

$$\Rightarrow g(g(x)) = \left(\frac{3}{2} - (g(x))^7\right)^{1/7} = \left(\frac{3}{2} - \left(\frac{3}{2} - x^7\right)\right)^{1/7} = x$$

Hence $g(g(100)) = 100$

16. (c) : $P(H) = P(T) = \dfrac{1}{2}$

$P(X = r) = {}^nC_r\, p^r q^{n-r}$

According to question,

$P(X = 1) + P(X = 3) = 2\,P(X = 2)$

ATTENTION COACHING INSTITUTES :

a great offer from MTG

CLASSROOM STUDY MATERIAL

MTG offers **"Classroom Study Material"** for JEE (Main & Advanced), NEET and FOUNDATION MATERIAL for Class 6, 7, 8, 9, 10, 11 & 12 **with YOUR BRAND NAME & COVER DESIGN.**

This study material will save your lots of money spent on teachers, typing, proof-reading and printing. Also, you will save enormous time. Normally, a good study material takes 2 years to develop. But you can have the material printed with your logo delivered at your doorstep.

Profit from associating with MTG Brand – the most popular name in educational publishing for JEE (Main & Advanced)/NEET....

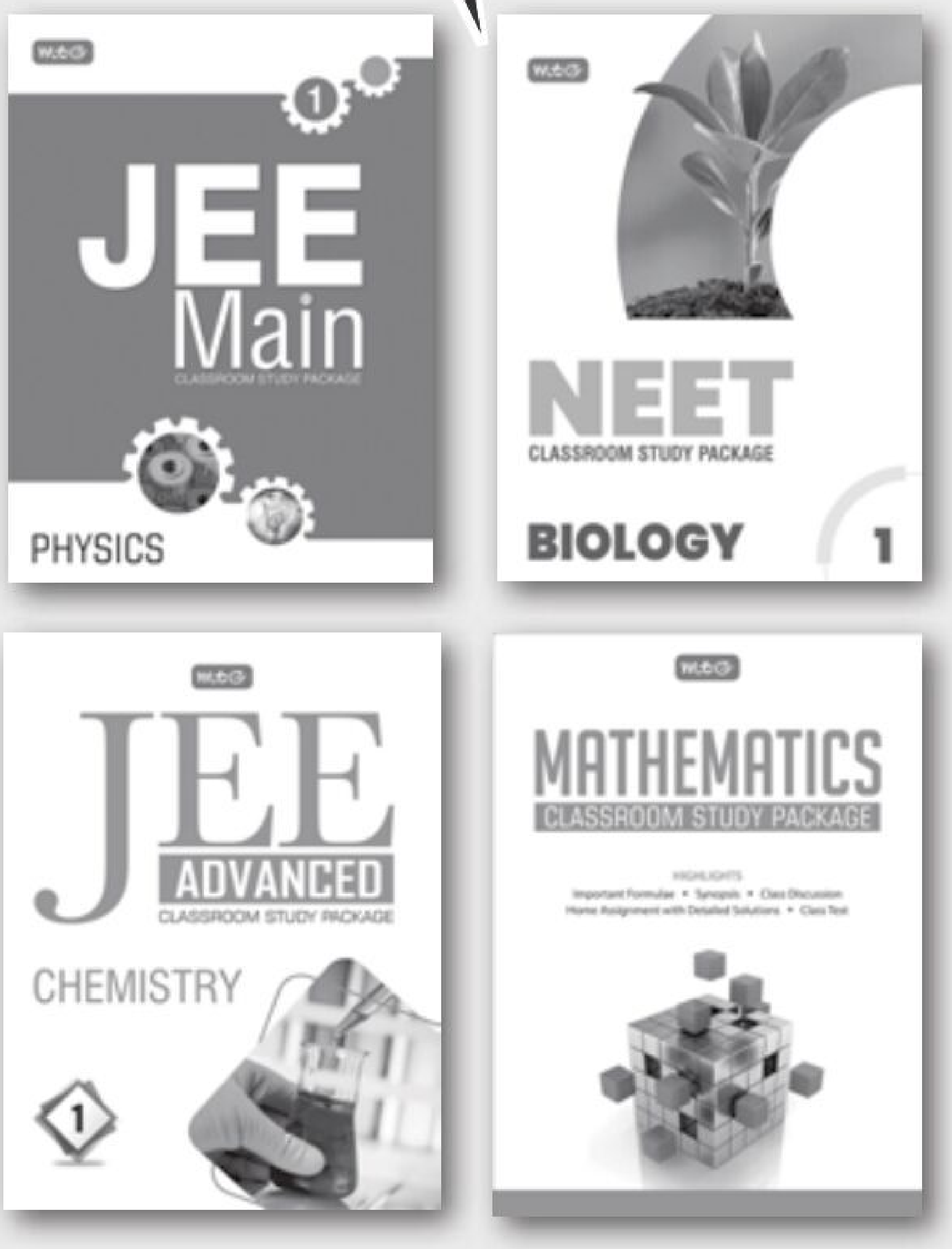
Order sample chapters on Phone/Fax/e-mail.

Phone : 0124-6601200 | 09312680856

e-mail : sales@mtg.in | www.mtg.in

★ EXCELLENT QUALITY ★

★ CONTENT ★ PAPER ★ PRINTING



$$\Rightarrow \; {}^nC_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{n-1} + {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3}$$

$$= 2\,{}^nC_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2}$$

$$\Rightarrow \; n + \frac{n(n-1)(n-2)}{6} = 2\,\frac{n(n-1)}{2}$$

$\Rightarrow$ $6 + (n-1)(n-2) = 6(n-1)$

$\Rightarrow$ $n^2 - 9n + 14 = 0 \Rightarrow n = 7$ or 2, But $n \neq 2$

$\therefore$ $n = 7$

17. (d) : $f(xy) = \cos \ln (xy) = \cos (\ln x + \ln y)$

Also, $f\left(\frac{x}{y}\right) = \cos \ln\left(\frac{x}{y}\right) = \cos (\ln x - \ln y)$

Now, $f(xy) + f\left(\frac{x}{y}\right) = \cos (\ln x + \ln y) + \cos (\ln x - \ln y)$

$= 2 \cos \ln x \cdot \cos \ln y = 2f(x)\, f(y)$

$$\therefore \; f(x)\, f(y) - \frac{1}{2}\left[f(xy) + f\left(\frac{x}{y}\right)\right] = 0$$

18. (b) : $\dfrac{x + (x-2)i}{3+i} + \dfrac{2y + i\,(1-3y)}{3-i} = i$

Multiplying by $(3+i)(3-i)$, we get

$(3-i)(x + (x-2)i) + (3+i)(2y + i(1-3y)) = 10i$

Equating the real parts on both sides

$3x + x - 2 + 6y - (1 - 3y) = 0$

$\Rightarrow$ $4x + 9y = 3$ ...(i)

Equating the imaginary parts, we get

$-x + 3(x-2) + 2y + 3(1-3y) = 10$

$\Rightarrow$ $2x - 7y = 13$ ...(ii)

Solving (i) and (ii), we get

$x = 3, y = -1, x + y = 2.$

19. (c) : $f(t) = t^3 - t^2 + t + 1$

$f'(t) = 3t^2 - 2t + 1 = 2t^2 + (t-1)^2 > 0$

$\therefore$ $f(t)$ is an increasing function.

$\therefore$ $g(x) = \max f(t) = f(x)$

$$\therefore \; g(x) = \begin{cases} x^3 - x^2 + x + 1, & 0 \le x \le 1 \\ 3 - x, & 1 < x \le 2 \end{cases}$$

$$g'(x) = \begin{cases} 3x^2 - 2x + 1, & 0 \le x \le 1 \\ -1, & 1 < x \le 2 \end{cases}$$

$g(1^-) = g(1^+) = g(1) = 2$

$\therefore$ $g(x)$ is continuous at $x = 1$

$g'(1^-) = 2, g'(1^+) = -1$

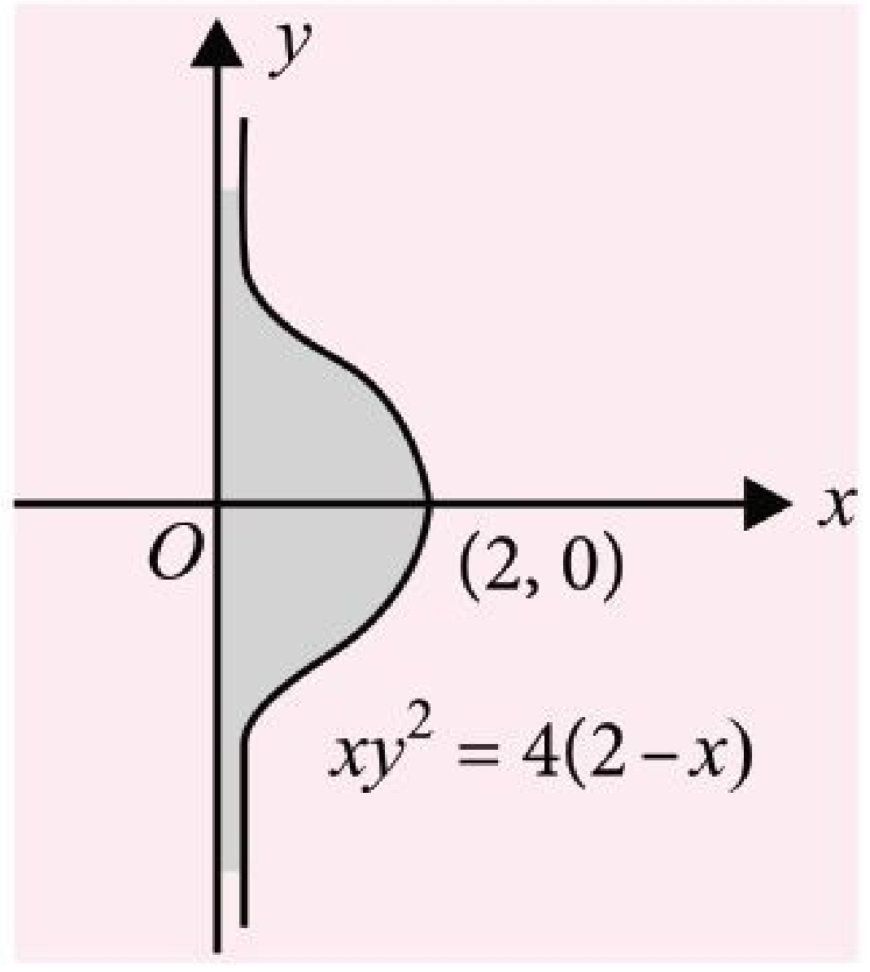
$\therefore$ $g(x)$ is not differentiable at $x = 1$.

20. (d) : $r_1 = 4R \sin\dfrac{A}{2} \cos\dfrac{B}{2} \cos\dfrac{C}{2}$,

$s = 4R \cos\dfrac{A}{2} \cos\dfrac{B}{2} \cos\dfrac{C}{2} \;\; \therefore \;\; \dfrac{r_1}{s} = \tan\dfrac{A}{2} = \dfrac{3}{4}$

$$\cos A = \frac{1 - \tan^2(A/2)}{1 + \tan^2 A/2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \frac{7}{25}$$

21. (1) : The curve $xy^2 = 4(2-x)$ is symmetrical about x-axis and at $x \to 0, y \to \infty$, *i.e.*, y-axis is asymptote and cuts x-axis at $x = 2$.



$\because \; y^2 = \dfrac{4(2-x)}{x} \ge 0$

$\Rightarrow \; \dfrac{2-x}{x} \ge 0 \Rightarrow \dfrac{(x-2)}{x} \le 0$

$\therefore \; x \in (0, 2)$

Then, $\lambda = 2\displaystyle\int_0^2 2\sqrt{\left(\frac{2-x}{x}\right)}dx$

Put $x = 2\sin^2\theta$, so that $dx = 4 \sin\theta \cos\theta\, d\theta$

$$\therefore \; \lambda = 4\int_0^{\pi/2} \sqrt{\left(\frac{2 - 2\sin^2\theta}{2\sin^2\theta}\right)} \times 4 \sin\theta \cos\theta\, d\theta$$

$$= 16\int_0^{\pi/2} \cos^2\theta\, d\theta = 16.\frac{1}{2}.\frac{\pi}{2} = 4\pi$$

Then, $\cos(1824\lambda) = \cos(1824 \times 4\pi) = 1$

22. (4) : $\dfrac{1}{y^3}\dfrac{dy}{dx} - \dfrac{x}{y^2} = x^3, z = \dfrac{1}{y^2} \Rightarrow \dfrac{dz}{dx} = \dfrac{-2}{y^3}\dfrac{dy}{dx}$

$\dfrac{dz}{dx} + 2xz = -2x^3$, linear equation

Solution is, $ze^{x^2} = -2\int x^3 \cdot e^{x^2} dx = (1 - x^2)e^{x^2} + A$

$\dfrac{1}{y^2} = 1 - x^2 + Ae^{-x^2}$, $y(0) = 1 \Rightarrow A = 0$

$\therefore \; y^2 = \dfrac{1}{1-x^2}, \; 3\left(y\left(\dfrac{1}{2}\right)\right)^2 = 4.$

23. (7) : The tangent with slope m to

$\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is $y = mx \pm \sqrt{16m^2 + 9}$

$\Rightarrow \; (mx - y)^2 = 16m^2 + 9 \Rightarrow (16 - x^2)m^2 + 2xym + 9 - y^2 = 0$

$\tan\theta_1 + \tan\theta_2 = \dfrac{-2xy}{16 - x^2}$,

$\tan\theta_1 \tan\theta_2 = \dfrac{9 - y^2}{16 - x^2}$

Now, $\theta_1 + \theta_2 = 2\alpha \Rightarrow \tan 2\alpha = \dfrac{-2xy}{7 - x^2 + y^2}$

$\Rightarrow\ 7 - x^2 + y^2 + 2xy\cot 2\alpha = 0$

$\Rightarrow\ x^2 - 2xy\cot 2\alpha - y^2 = 7.\ \therefore\ \lambda = 7$

24. (1) : $f = g^{-1}$ *i.e.* $g^{-1}(x) = f(x)$
where $g(x) = y = 2\tan^{-1}(e^x) - \pi/2$

$$\Rightarrow \tan^{-1}(e^x) = \frac{\pi}{4} + \frac{y}{2}\ \Rightarrow\ x = \log\tan\left(\frac{\pi}{4} + \frac{y}{2}\right)$$

$$\Rightarrow\ g^{-1}(y) = \log\ \tan\left(\frac{\pi}{4} + \frac{y}{2}\right)$$

$$\Rightarrow\ g^{-1}(x) = f(x) = \log\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$$

$$\Rightarrow\ f'(x) = \frac{1}{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)}\cdot\sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right)\times\frac{1}{2}$$

$$f'(0) = \frac{1}{\tan\frac{\pi}{4}}\times\sec^2\frac{\pi}{4}\times\frac{1}{2} = 1$$

25. (5) : $(1 + x(2 + 3x))^{10}$

$$= 1 + \binom{10}{1}x(2+3x) + \binom{10}{2}x^2(2+3x)^2 + \binom{10}{3}x^3(2+3x)^3 + \binom{10}{4}x^4(2+3x)^4 + \ldots$$

$\therefore$ Coefficient of x^4 is

$$\binom{10}{2}\cdot 3^2 + \binom{10}{3}\cdot 3\cdot 2^2\cdot 3 + \binom{10}{4}\cdot 2^4$$

$= 405 + 4320 + 3360 = 8085.$

26. (6) : The plane containing the lines
$\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \mu\vec{d}$ is $[\vec{r} - \vec{a}\quad \vec{b}\quad \vec{d}] = 0$

$$\therefore \begin{vmatrix} x-1 & y-2 & z-3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0\ \Rightarrow\ x - 2y + z = 0 \qquad \text{...(i)}$$

$Ax - 2y + z = d$ is parallel to (i)

$\therefore\ A = 1$. Now the planes are $x - 2y + z = 0$, $x - 2y + z = d$

Distance between these parallel planes is

$$\frac{|d|}{\sqrt{1^2 + 2^2 + 1^2}} = \sqrt{6}\ \text{(given)} \qquad \therefore\ |d| = 6.$$

27. (0) : $ax^2 + 2cx + b = 0$ ...(i)

$ax^2 + 2bx + c = 0$ ...(ii)

(i) and (ii) imply that

$$\frac{x^2}{2(c^2 - b^2)} = \frac{x}{a(b-c)} = \frac{1}{2a(b-c)}$$

$\Rightarrow\ 4a(c^2 - b^2)(b - c) = a^2(b - c)^2$

$\Rightarrow\ -4(b + c) = a\ \Rightarrow\ a + 4b + 4c = 0.$

28. (5) : $(A + B)^2 = A^2 + B^2$

$\Rightarrow\ (A + B)(A + B) = A^2 + B^2$

$\Rightarrow\ A^2 + BA + AB + B^2 = A^2 + B^2 \Rightarrow AB + BA = O$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} + \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a-b+2 & -a+1 \\ 2a-2 & -b+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow\ 2a - b + 2 = 0,\ -a + 1 = 0,\ 2a - 2 = 0,\ -b + 4 = 0$

$\Rightarrow\ a = 1, b = 4\ \therefore\ a + b = 1 + 4 = 5$

29. (1) : Let m, f and s denote the mother, the father and the son respectively then sample space S is

$S = \{mfs, msf, fms, fsm, smf, sfm\}$

Let E : 'son on one end' and F : 'father in the middle',
i.e., $E = \{mfs, fms, smf, sfm\}$ and $F = \{sfm, mfs\}$

$\Rightarrow E \cap F = \{mfs, sfm\}$

$$P(E) = \frac{4}{6} = \frac{2}{3}, P(F) = \frac{2}{6} = \frac{1}{3},\ P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore\ \ P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1/3}{1/3} = 1.$$

30. (9) : $\tan(19x)^\circ = \dfrac{\cos 96^\circ + \cos 6^\circ}{\cos 96^\circ - \cos 6^\circ} = \dfrac{\cos 51^\circ}{-\sin 51^\circ}$

$$\tan(19x)^\circ = \frac{\cos 96^\circ + \cos 6^\circ}{\cos 96^\circ - \cos 6^\circ} = \frac{\cos 51^\circ}{-\sin 51^\circ}$$

$\therefore\ \ 19x = 180n - 39$

$= 190n - 38 - (10n + 1)$

$$x = 10n - 2 - \frac{10n+1}{19}$$

The least term is an integer for $n = 17$

$\therefore\ \ \alpha = 170 - 2 - 9 = 159$

◆

Your favourite MTG Books/Magazines available in
UTTARAKHAND at

National Book House - Dehradun Ph: 0135-2659430; 9897830283
Army Book Depot - Dehradun Ph: 0135-2756683; 9897927902
Om Vidya Educational Book - Dehradun Ph: 9897833882
Career Zone - Haldwani Ph: 05946-262051; 9412128075, 94123833167
World Vision Publication - Haldwani Ph: 9927932200, 9027240169
Diamond Stationers - Haridwar Ph: 0133-4252043; 9358398035, 9359763348
Consul Book Depot - Nainital Ph: 0592-235164; 9412084105
Indra Book Emporium - Roorkee Ph: 0133-276105; 8126293314
Cambridge Book Depot - Roorkee Ph: 272341, 272345; 9719190955

Visit **"MTG IN YOUR CITY"** on www.mtg.in to locate nearest book seller OR write to **info@mtg.in** OR call **0124-6601200** for further assistance.

JEE 2022 PRACTICE PAPER ADVANCED

Exam on 28th August

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. A die is thrown six times, the chance that exactly 3 times an odd number turn up is

(a) $\frac{1}{16}$ (b) $\frac{3}{16}$

(c) $\frac{5}{16}$ (d) None of these

2. The solution set of $x^2 + 2 \le 3x \le 2x^2 - 5$ is

(a) ϕ (b) $[1, 2]$

(c) $(-\infty, -1) \cup \left[\frac{5}{2}, +\infty\right)$

(d) None of the above

3. If the number of ways of selecting 3 numbers out of 1, 2, 3, ..., $2n + 1$ numbers such that they are in A.P. is 441, then the sum of divisors of n is

(a) 21 (b) 22

(c) 32 (d) None of these

4. If $a = \lim\limits_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$, then

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

is equal to

(a) $\frac{1}{a}$ (b) $2a$

(c) $\frac{a}{2}$ (d) $\frac{3}{8}$

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

Consider the letters of the word MATHEMATICS.

5. Possible number of words taking all letters at a time such that atleast one repeating letter is at odd position in each word is $\frac{a!}{(b!)^3}$, then $a + b =$ _______.

6. Possible number of words taking all letters at a time such that in each word both M's are together and both T's are together but both A's are not together is $a! \times {}^bC_2$, where $ab =$ _______.

Question Stem for Question Nos. 7 and 8

Consider the function $f(x)$ such that

$$f(x) = x^2 + \int_0^1 (x + t) f(t)\, dt$$

7. The area bounded by the x-axis, the tangent and normal at $x = 0$ on the curve $y = f(x)$ is $\frac{a}{b}$, then, $a - 20b$ = _______.

8. The area bounded by the curve $y = f(x)$ and the x-axis is $\frac{a}{b}\sqrt{\frac{a}{3}}$, then $a =$ _______.

Question Stem for Question Nos. 9 and 10

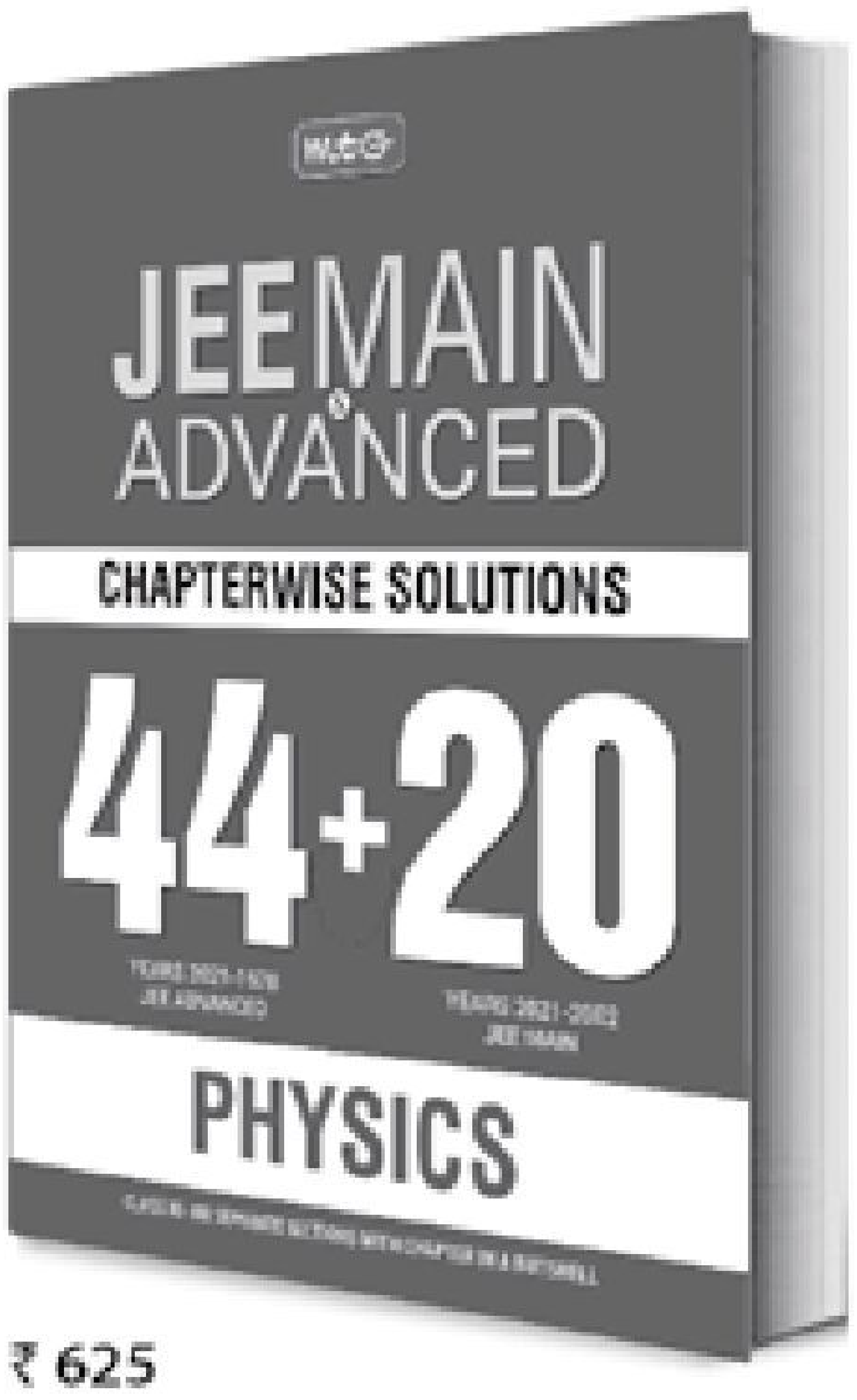
An equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$, is called a linear differential equation of the first order. Its solution is given as $ye^{\int P dx} = \int Q e^{\int P dx} + k$, k being the constant of integration. A number of differential equations can be reduced to the linear differential equation and then one can solve them.

9. The general solution of the differential equation $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$ (c being the constant of integration) is $e^y = e^x - k + ce^{-e^x}$, then $k =$

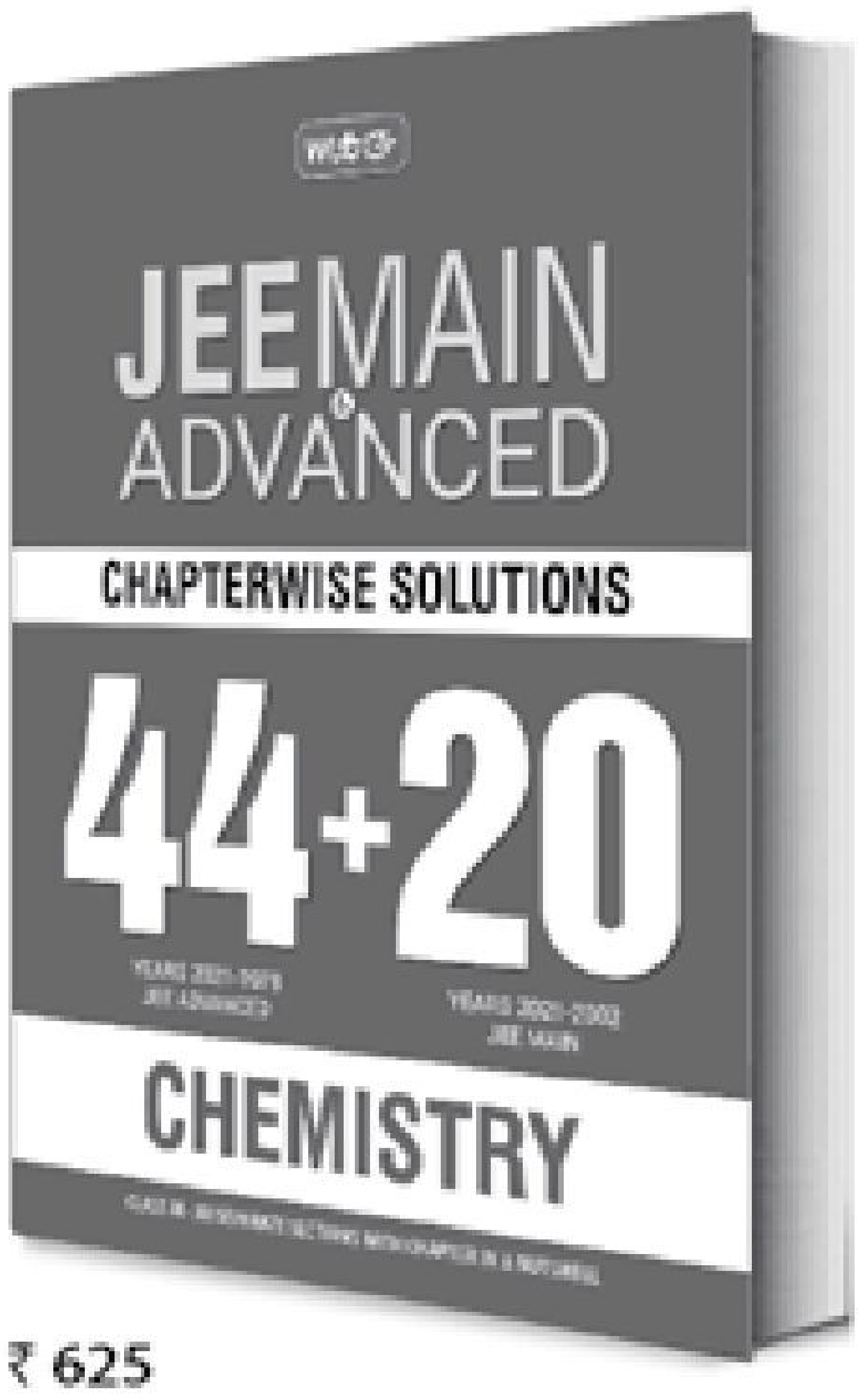
mtG

Some of the best lessons are learnt from history!

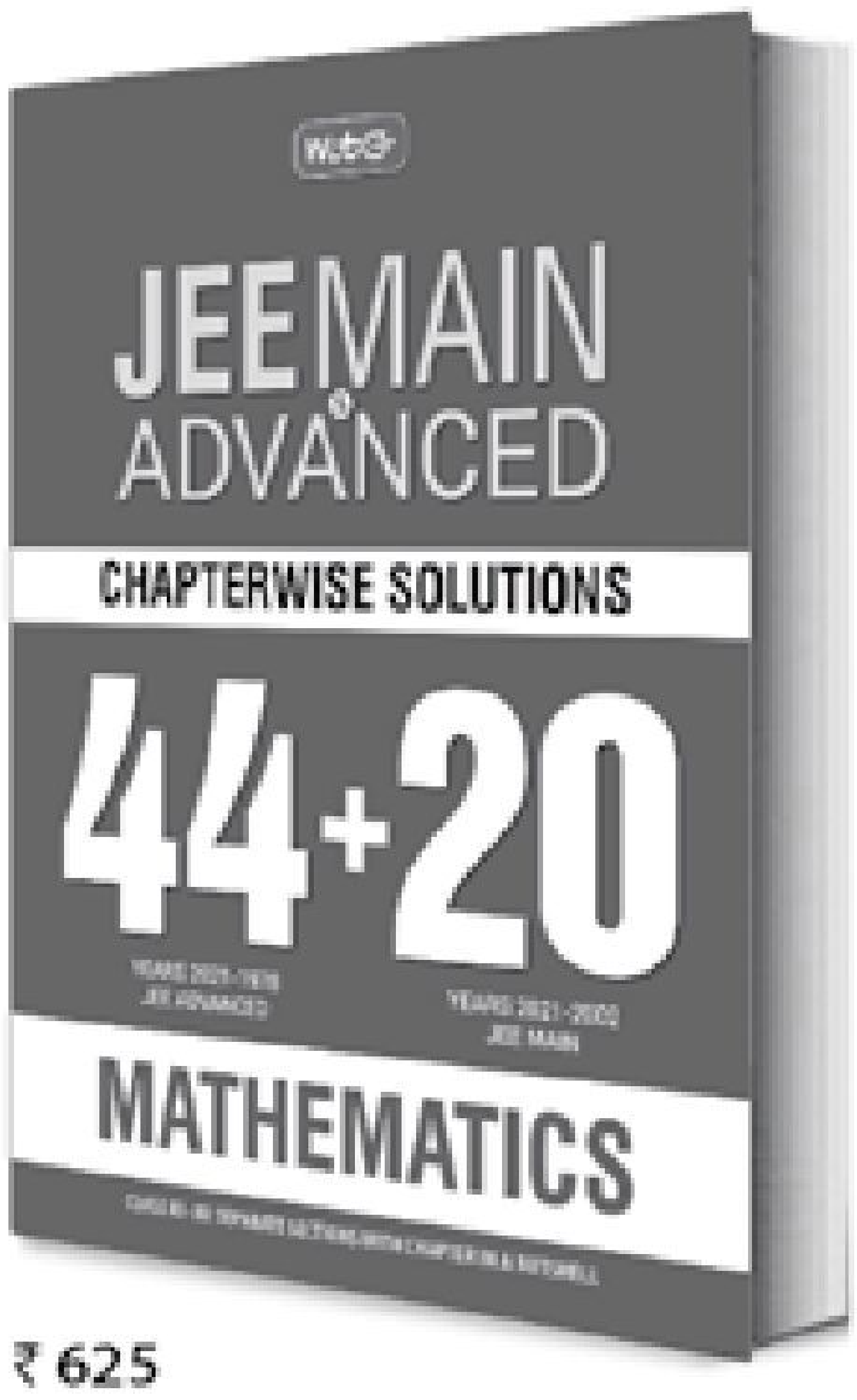
Introducing Chapterwise Solutions to JEE.
Crack the past to see if you will last.


mtG
JEE MAIN & ADVANCED
CHAPTERWISE SOLUTIONS
44+20
PHYSICS

₹ 625


mtG
JEE MAIN & ADVANCED
CHAPTERWISE SOLUTIONS
44+20
CHEMISTRY

₹ 625


mtG
JEE MAIN & ADVANCED
CHAPTERWISE SOLUTIONS
44+20
MATHEMATICS

₹ 625

Tune. Fine tune. Reach the peak of your readiness for JEE with MTG's 44+20 Years Chapterwise Solutions. It is undoubtedly the most comprehensive 'real' question bank, complete with detailed solutions by experts.

Studies have shown that successful JEE aspirants begin by familiarising themselves with the problems that have appeared in past JEEs as early as 2 years in advance.

Making it one of the key ingredients for their success. How about you then?
Get 44+20 Years Chapterwise Solutions to start your rehearsals early.

mtG
Learning Media

Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Visit **www.mtg.in** to buy online!
Call 0124-6601200 or e-mail:info@mtg.in

10. The general solution of the differential equation $\frac{dy}{dx}+\frac{y\ln y}{x}=\frac{y}{x^2}(\ln y)^2$ (c being the constant of integration) is $ax = (1 + acx^2)\ln y$, where $a =$

SECTION 3

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

11. A continuous and differentiable function f satisfies the condition, $\int_0^x f(t)dt = f^2(x) - 1$ for all real x. Then

(a) f is monotonic increasing $\forall\ x \in R$
(b) f is monotonic decreasing $\forall\ x \in R$
(c) f is non monotonic
(d) the graph of $y = f(x)$ is a straight line

12. If $(\sqrt{2})^x + (\sqrt{3})^x = (\sqrt{13})^{x/2}$, then the value of x is a/an

(a) negative integer (b) irrational number
(c) imaginary number (d) natural number

13. If the hyperbola $xy = c^2$ intersects the circle $x^2 + y^2 = a^2$ at four points $P_i(x_i, y_i)$, $i = 1, 2, 3, 4$, then

(a) $x_1 + x_2 + x_3 + x_4 = 0$ (b) $y_1 + y_2 + y_3 + y_4 = 0$
(c) $x_1x_2x_3x_4 = c^4$ (d) $y_1y_2y_3y_4 = c^4$

14. If a, b, c are in A.P. and a^2, b^2, c^2 are in H.P., then

(a) $a = b = c$ (b) $a^2 = b^2 = \frac{c^2}{2}$
(c) a, b, c are in G.P. (d) $-\frac{a}{2}, b, c$ are in G.P.

15. Let $\vec{a}=\begin{bmatrix}1\\0\\-3\end{bmatrix}, \vec{b}=\begin{bmatrix}2\\1\\0\end{bmatrix}, \vec{c}=\begin{bmatrix}1\\-1\\1\end{bmatrix}$. If the numbers α, β and γ are such that $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=\begin{bmatrix}-2\\-5\\6\end{bmatrix}$, then

(a) $\alpha = -1$ (b) $\beta = -2$
(c) $\gamma = 3$ (d) $\alpha + \beta + \gamma = 0$

16. For a positive integer n, if the expansion of $\left(\frac{5}{x^2}+x^4\right)^n$ has a term independent of x, then n can be

(a) 18 (b) 21 (c) 27 (d) 99

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. The distance of the point (3, 0, 5) from the line $x - 2y + 2z - 4 = 0 = x + 3z - 11$ is

18. Consider a triangle ABC, if $a = 6$, $b = 10$, $\Delta = 15\sqrt{3}$ and $\angle C$ is obtuse, then r^2 is _______ .

19. The area of the largest circle with centre $\left(\frac{1}{2}, 0\right)$ inscribed in the ellipse $x^2 + 2y^2 = 2$ is $\frac{m\pi}{n}$, where m and n are relatively prime positive integers. Then $m + n$ is

PAPER-2

SECTION 1

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

1. The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking it was found that an observation 8 was incorrect.

Then the correct standard deviation when the wrong item is omitted is

(a) 1.997 (b) 2
(c) 10.15 (d) 3

2. The roots of equation $x^{[(\log_3 x)^2-(9/2)\log_3 x+5]} = 3\sqrt{3}$ are

(a) $\sqrt{3}$ (b) 9 (c) 3 (d) 27

3. If $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P., then $a, \frac{1}{b}$, are in

(a) A.P. (b) G.P.
(c) H.P. (d) None of these

4. If $z^3 + (3 + 2i)z + (-1 + ia) = 0$ has one real root, then the value of a lies in the interval ($a \in R$)

(a) $(-2, 1)$ (b) $(-1, 0)$ (c) $(0, 1)$ (d) $(-2, 3)$

5. The equation of a hyperbola with co-ordinate axes as principal axes, and the distances of one of its vertices from the foci are 3 and 1 can be

(a) $3x^2 - y^2 = 3$ (b) $x^2 - 3y^2 - 3 = 0$
(c) $x^2 - 3y^2 + 1 = 0$ (d) none of these

6. $\sin^{-1} x > \cos^{-1} x$ holds for

(a) all values of x (b) $x \in \left(0, \frac{1}{\sqrt{2}}\right)$
(c) $x \in \left(\frac{1}{\sqrt{2}}, 1\right)$ (d) $x = 0.75$

SECTION 2

- *This section contains THREE (03) question stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 7 and 8

If an equilateral triangle is inscribed in the circle, $|z| = 2$ and if $z_1 = 1 + \sqrt{3}i$, then

7. If value of z_2 equals $-n$, then $n =$

8. If value of z_3 equals $a - \sqrt{3}i$, then $a =$

Question Stem for Question Nos. 9 and 10

Let ABC be a triangle with $A(-3, 1)$. The equation of the median through B is $2x + y = 3$ and the equation of the angular bisector of C is $7x - 4y = 1$.

9. The distance of the origin from the side AC is $a/\sqrt{3}$, where $a =$

10. The distance of the origin from the side BC is $\frac{b}{5\sqrt{13}}$, where $b =$

Question Stem for Question Nos. 11 and 12

If $\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix} = 0$, $x \neq a, y \neq b, z \neq c$ then

11. The value of $-\left(\frac{x+a}{x-a} + \frac{y+b}{y-b} + \frac{z+c}{z-c}\right)$ is equal to

12. The value of $\frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c}$ is equal to

SECTION 3

- *This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO(02) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the option corresponding to the correct answer.*

Paragraph for Q. 13 and Q. 14

In a triangle if the sum of two sides is x and this product is $y\,(x \geq 2\sqrt{y})$ such that $(x + z)(x - z) = y$, where z is the third side of the triangle.

13. Greatest angle of the triangle is

(a) 105° (b) 120° (c) 135° (d) 150°

14. Circumradius of the triangle is

(a) x (b) y (c) z (d) $z/\sqrt{3}$

Paragraph for Q. 15 and Q. 16

In a triangle ABC, let $r_1 = 39$, $r_2 = 70$ and $\angle C = \pi/2$.

15. The area of Δ is the product of n primes, where $n =$

(a) 3 (b) 4 (c) 5 (d) 6

16. The value of $c =$

(a) 109 (b) 109/2 (c) 218 (d) 79

SECTION 4

- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*

17. The sum to 20 terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \ldots\ldots$ is ______ .

18. The number of solutions of the equations $x + 4y - z = 0$, $3x - 4y - z = 0$, $x - 3y + z = 0$ is ______.

19. If $\int_0^{\frac{\pi}{2010}} \frac{dx}{(\cot 1005x)^{1005} + 1} = \frac{\pi}{4K}$, then $K =$

SOLUTIONS

PAPER-1

1. (c) : p = Probability of success = 3/6 = 1/2

q = Probability of failure = 1 – 1/2 = 1/2

$\therefore \quad P(X = r) = {}^nC_r\, p^r q^{n-r}$

$$P(X = 3) = {}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{6-3} = {}^6C_3\,\frac{1}{64} = \frac{5}{16}$$

2. (a)

3. (a) : Let the three selected numbers in A.P. be a, b, c then $b = \frac{a+c}{2}$

Since, b is an integer, therefore, we have to select only a and c such that both of then are odd or both are even.

$\therefore \quad$ Number of ways $= {}^{n+1}C_2 + {}^nC_2 = n^2$

Given $n^2 = 441 \Rightarrow n = 21 = 3 \times 7$

Sum of divisors of n *i.e.*, $21 = (1 + 3)(1 + 7) = 32$

4. (a) : Given, $a = \lim\limits_{x \to 2} \frac{2^x + 2^{3-x} - 6}{\sqrt{2^{-x}} - 2^{1-x}}$

$$= \lim_{y \to 4} \frac{y + \frac{8}{y} - 6}{\frac{1}{\sqrt{y}} - \frac{2}{y}}, \text{ where } y = 2^x = \lim_{y \to 4} \frac{y^2 - 6y + 8}{\sqrt{y} - 2}$$

$$= \lim_{y \to 4} \frac{(y-2)(\sqrt{y}+2)(\sqrt{y}-2)}{\sqrt{y}-2} = 8$$

Now, $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$=\left(\sin\frac{\pi}{8}\sin\frac{3\pi}{8}\right)^2 = \frac{1}{4}\left(2\sin\frac{3\pi}{8}\sin\frac{\pi}{8}\right)^2$$

$$=\frac{1}{4}\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right)^2 = \frac{1}{8} = \frac{1}{a}$$

5. (13) : Since there are five even places and three pairs of repeated letters. Therefore, atleast one of these must be at an odd place. Therefore, the number of ways is $\frac{11!}{(2!2!2!)}$.

6. (56) : Make a group of both M's and another group T's. Then except A's we have five letters remaining. So, M's, T's and the letters except A's can be arranged in 7! ways. Therefore, total number of arrangements is $7! \times {}^8C_2 = a! \times {}^bC_2$.

$\therefore \quad ab = 56$

7. (157) : Let $y(x) = x^2 + ax + b$, where $a = \int_0^1 y(t)\,dt$, $b = \int_0^1 t\,y(t)\,dt$ $\therefore \quad a = \frac{1}{3}+\frac{a}{2}+b,\ b = \frac{1}{4}+\frac{a}{3}+\frac{b}{2}$

$$\Rightarrow \frac{a}{2}-b = \frac{1}{3},\ \frac{b}{2}-\frac{a}{3} = \frac{1}{4}$$

By solving, we get, $a = -5,\ b = -\frac{17}{6}$

$\therefore\ f(x) = x^2 - 5x - \frac{17}{6} \Rightarrow f'(x) = 2x - 5$

Tangent at $x = 0$ is $y + \frac{17}{6} = -5x$ which meets the x-axis at $\left(\frac{-17}{30}, 0\right)$

The normal at $x = 0$ is $y + \frac{17}{6} = \frac{x}{5}$ which meets the x-axis at $\left(\frac{85}{6}, 0\right)$

$$\text{Area} = \frac{1}{2}\left(\frac{17}{6}\right)\left(\frac{85}{6}+\frac{17}{30}\right) = \frac{3757}{180}$$

8. (109) : $x^2 - 5x - \frac{17}{6} = 0$ gives the roots

$$\alpha = \frac{5}{2} - \frac{1}{2}\sqrt{\frac{109}{3}},\ \beta = \frac{5}{2} + \frac{1}{2}\sqrt{\frac{109}{3}}$$

$\therefore$ Required area, $A = -\int_\alpha^\beta f(x)\,dx$

$$= -\int_\alpha^\beta \left(x^2 - 5x - \frac{17}{6}\right)dx = -\left[\frac{x^3}{3} - \frac{5x^2}{2} - \frac{17}{6}x\right]_\alpha^\beta$$

$$= \frac{-(\beta-\alpha)}{6}[2(\alpha^2+\beta^2+\alpha\beta) - 15(\alpha+\beta) - 17]$$

$$= \frac{-(\beta-\alpha)}{6}[2(\alpha+\beta)^2 - 2\alpha\beta - 15(\alpha+\beta) - 17]$$

But $\alpha + \beta = 5$, $\alpha\beta = \frac{-17}{6}$

$$\therefore\ A = \frac{-1}{6}\sqrt{\frac{109}{3}}\left[50 + \frac{17}{3} - 75 - 17\right] = \frac{109}{18}\sqrt{\frac{109}{3}}$$

9. (1) : We have, $\frac{dy}{dx} = e^{x-y}(e^x - e^y)$

$$e^y\frac{dy}{dx} = e^{2x} - e^{x+y} \Rightarrow e^y\frac{dy}{dx} + e^x e^y = e^{2x}$$

Let $e^y = u$, so that $e^y\frac{dy}{dx} = \frac{du}{dx}$

We have, $\frac{du}{dx} + e^x \cdot u = e^{2x}$, then

$\text{I.F.} = e^{\int e^x dx = e^{e^x}}$ $\quad\therefore$ Solution is $u \cdot e^{e^x} = \int e^{e^x} . e^{2x} dx$

$= \int e^t \cdot t\,dt$, where $t = e^x$

$= e^t(t-1) + c \quad = e^{e^x}(e^x - 1) + c$

Then $u = e^x - 1 + ce^{-e^x} \Rightarrow e^y = e^x - 1 + c\,e^{-e^x}$

10. (2) : We have, $\frac{dy}{dx} + \frac{y\ln y}{x} = \frac{y}{x^2}(\ln y)^2$

$$\Rightarrow \frac{1}{y(\ln y)^2}\frac{dy}{dx} + \frac{1}{x}.\frac{1}{\ln y} = \frac{1}{x^2}$$

Let $\frac{-1}{\ln y} = u$, so that $\frac{du}{dx} - \frac{u}{x} = \frac{1}{x^2}$

$$\text{I.F.} = e^{-\int\frac{dx}{x}} = e^{-\ln x} = \frac{1}{x}$$

Then the solution is $u.\frac{1}{x}=\int\frac{1}{x^3}dx+c,$

which on simplification gives $2x = (1 + 2cx^2)\ln y$

11. (a,d) : On Differentiating, $f(x) = 2\,f(x)\cdot f'(x)$

$$\Rightarrow f'(x)=\frac{1}{2},(f(x)\neq 0) \qquad \therefore\ f(x)=\frac{x}{2}+c$$

which is a straight line.

12. (d) : Given, $(\sqrt{2})^x+(\sqrt{3})^x=(\sqrt{13})^{x/2}$

$$\Rightarrow \frac{(2)^{x/2}}{(\sqrt{13})^{x/2}}+\frac{(3)^{x/2}}{(\sqrt{13})^{x/2}}=1 \Rightarrow \left(\frac{2}{\sqrt{13}}\right)^{x/2}+\left(\frac{3}{\sqrt{13}}\right)^{x/2}=1$$

Let $2 = r\sin\theta$, $3 = r\cos\theta \Rightarrow r=\sqrt{13}$

$$\Rightarrow \sin^{\frac{x}{2}}\theta+\cos^{\frac{x}{2}}\theta = 1 = \sin^2\theta+\cos^2\theta$$

$$\Rightarrow \frac{x}{2}=2 \Rightarrow x=4$$

13. (a, b, c, d) : The point $\left(ct,\frac{c}{t}\right)$ lies on $x^2 + y^2 = a^2$

$$\therefore\ c^2t^2+\frac{c^2}{t^2}=a^2 \text{ or } c^2t^4 - a^2t^2 + c^2 = 1$$

If t_1, t_2, t_3, t_4 are the roots, then

$$t_1+t_2+t_3+t_4=0, t_1t_2t_3t_4=\frac{c^2}{c^2}=1, \qquad \text{...(i)}$$

$$t_1t_2t_3 + t_2t_3t_4 + t_3t_4t_1 + t_4t_1t_2 = 0 \qquad \text{...(ii)}$$

$$\text{(i), (ii)} \Rightarrow \frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\frac{1}{t_4}=0 \qquad \text{...(iii)}$$

$$\therefore\ x_1 + x_2 + x_3 + x_4 = c(t_1 + t_2 + t_3 + t_4) = 0$$

$$y_1+y_2+y_3+y_4=c\left(\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\frac{1}{t_4}\right)=0$$

$$x_1\,x_2\,x_3\,x_4 = c^4\,t_1\,t_2\,t_3\,t_4 = c^4$$

and $y_1\,y_2\,y_3\,y_4 = \frac{c^4}{t_1t_2t_3t_4}=c^4.$

14. (a,c,d) : From the given conditions, we have

$2b = a + c$ and $\frac{2}{b^2}=\frac{1}{a^2}+\frac{1}{c^2}$

$\Rightarrow\ 2a^2c^2 = b^2(a^2 + c^2)$

Eliminating b, we get

$$2a^2c^2=\frac{1}{4}(a+c)^2\,(a^2+c^2)$$

$\Rightarrow\ 8a^2c^2 = (a^2 + c^2 + 2ac)\,(a^2 + c^2)$

$\Rightarrow\ (a^2 + c^2)2 + 2ac\,(a^2 + c^2) - 8a^2c^2 = 0$

$\Rightarrow\ (a^2 + c^2 - 2ac)\,(a^2 + c^2 + 4ac) = 0$

$\Rightarrow\ a^2 + c^2 - 2ac = 0$ or $a^2 + c^2 + 4ac = 0$

$\Rightarrow\ (a - c)^2 = 0$ or $(a + c)^2 + 2ac = 0$

Hence, $a = c$ or $4b^2 + 2ac = 0$ *i.e.*, $b^2=-\frac{1}{2}ac$.

If $a = c$, then from $2b = a + c$, we get $b = a$, so that $a = b = c$, in which case a, b and c are evidently in G.P.

If $b^2=-\frac{1}{2}ac$, then $-\frac{a}{2}, b$ and c are in G.P.

15. (a, b, c, d) : We have

$$\alpha(\hat{i}-3\hat{k})+\beta(2\hat{i}+\hat{j})+\gamma(\hat{i}-\hat{j}+\hat{k})=-2\hat{i}-5\hat{j}+6\hat{k}$$

On comparing, we get

$$\alpha + 2\beta + \gamma = -2 \qquad \text{...(i)}$$

$$\beta - \gamma = -5 \qquad \text{...(ii)}$$

$$-3\alpha + \gamma = 6 \qquad \text{...(iii)}$$

PUBLIC NOTICE **JEE ADVANCED 2022**

S. No.	Activity	Day, Date and Time (IST)
1.	JEE Main 2022 (Computer based test by NTA)	Please refer to JEE (Main) 2022 website
2.	Results JEE Main 2022 from NTA	Please refer to JEE (Main) 2022 website
3.	Registration for JEE (Advanced) 2022	Sunday, August 07, 2022 (10:00 IST) to Thursday, August 11, 2022 (17:00 IST)
4.	Last date for fee payment of registered candidates	Friday, August 12, 2022 (17:00 IST)
5.	Admit Card available for downloading	Tuesday, August 23, 2022 (10:00 IST) to Sunday, August 28, 2022 (14:30 IST)
6.	Choosing of scribe by PwD candidates	Saturday, August 27, 2022
7.	**JEE (Advanced) 2022**	**Sunday, August 28, 2022** **Paper 1: 09:00-12:00 IST** **Paper 2: 14:30-17:30 IST**
8.	Copy of candidate responses to be available on the JEE (Advanced) 2022 website	Thursday, September 01, 2022 (10:00 IST)
9.	Online display of provisional answer keys	Saturday, September 03, 2022 (10:00 IST)
10.	Feedback and comments on provisional answer keys from the candidates	Saturday, September 03, 2022 (10:00 IST) to Sunday, September 04, 2022 (17:00 IST)
11.	Online declaration of final answer keys	Sunday, September 11, 2022 (10:00 IST)
12.	**Result of JEE (Advanced) 2022**	**Sunday, September 11, 2022 (10:00 IST)**
13.	Online registration for Architecture Aptitude Test (AAT) 2022	Sunday, September 11, 2022 (10:00 IST) to Monday, September 12, 2022 (17:00 IST)
14.	Tentative Start of Joint Seat Allocation (JoSAA) 2022 Process	Monday, September 12, 2022
15.	Architecture Aptitude Test (AAT) 2022	Wednesday, September 14, 2022 (09:00-12:00 IST)
16.	Declaration of results of AAT 2022	Saturday, September 17, 2022 (17:00 IST)

For more information visit jeeadv.ac.in

Solving the equations (i), (ii) and (iii), we get
$\alpha = -1, \beta = -2, \gamma = 3$

16. (a, b, c, d) : Let $(r+1)^{\text{th}}$ term of $\left(\frac{5}{x^2} + x^4\right)^n$ be independent of x.

$$\therefore \quad T_{r+1} = {}^nC_r\left(\frac{5}{x^2}\right)^{n-r}\left(x^4\right)^r = {}^nC_r 5^{n-r}x^{6r-2n}$$

For this term to be independent of x, $6r - 2n = 0$
$\Rightarrow \quad n = 3r$, where $r = 0, 1, 2, \ldots, n$.

17. (3) : The d.r's of the line are given by

$$l - 2m + 2n = 0,\ l + 3n = 0 \Rightarrow \frac{l}{6} = \frac{m}{1} = \frac{n}{-2}$$

Taking $y = 0$, we get $x + 2z = 4$, $x + 3z = 11$
$\Rightarrow x = -10, z = 7$

The line is $\frac{x+10}{6} = \frac{y}{1} = \frac{z-7}{-2}$; $\vec{b} = 6\hat{i} + \hat{j} - 2\hat{k}$

$\therefore$ Distance of C (3, 0, 5)from the line is $\frac{|\overrightarrow{AC} \times \vec{b}|}{|\vec{b}|}$

$$= \frac{|(13\hat{i} - 2\hat{k}) \times (6\hat{i} + \hat{j} - 2\hat{k})|}{\sqrt{41}}$$

$$= \frac{|2\hat{i} + 14\hat{j} + 13\hat{k}|}{\sqrt{41}} = \frac{\sqrt{369}}{\sqrt{41}} = 3.$$

18. (3) : $\Delta = \frac{1}{2}ab \sin C$

$$\Rightarrow \quad \sin C = \frac{2\Delta}{ab} = \frac{30\sqrt{3}}{60} = \frac{\sqrt{3}}{2} \Rightarrow C = \frac{2\pi}{3}$$

Now, $c^2 = a^2 + b^2 - 2ab\cos C = 136 + 60 = 196$
$\Rightarrow \quad c = 14$

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{6+10+14}{2} = 15$$

Now, $\Delta = rs \Rightarrow r = \frac{15\sqrt{3}}{15} = \sqrt{3}$

19. (7) : If r is the radius of the circle and $P(\sqrt{2}\cos\theta, \sin\theta)$ is a point on the ellipse,

$$r^2 = \left(\sqrt{2}\cos\theta - \frac{1}{2}\right)^2 + \sin^2\theta = \frac{3}{4} + \left(\cos\theta - \frac{1}{\sqrt{2}}\right)^2 \geq \frac{3}{4}$$

The largest radius $r = \frac{\sqrt{3}}{2}$, Area $= \frac{3}{4}\pi = \frac{m\pi}{n}$
$\therefore \quad m + n = 3 + 4 = 7.$

PAPER-2

1. (a) : Mean $= 10 \Rightarrow \frac{\sum_{i=1}^{20} x_i}{20} = 10 \Rightarrow \sum_{i=1}^{20} x_i = 200$

and S.D. $= 2 \Rightarrow \sigma^2 = 4$

$$\Rightarrow \frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4 \Rightarrow \sum_{i=1}^{20} x_i^2 = 2080$$

Thus, incorrect $\left(\sum_{i=1}^{20} x_i\right) = 200$ and incorrect $\left(\sum_{i=1}^{20} x_i^2\right) = 2080$

On omitting 8, we are left with 19 observations.

$$\therefore \quad \text{Correct}\left(\sum_{i=1}^{19} x_i\right) = \text{Incorrect}\left(\sum_{i=1}^{20} x_i\right) -$$

$= (200 - 8) = 192$

$$\therefore \quad \text{Correct mean} = \frac{192}{19}$$

$$\text{Also, correct}\left(\sum_{i=1}^{19} x_i^2\right) = \text{incorrect}\left(\sum_{i=1}^{20} x_i^2\right) - 64$$

$= (2080 - 64) = 2016$

$$\therefore \quad \text{Correct S.D.} = \sqrt{\frac{1}{19}\left(\text{correct}\sum_{i=1}^{19} x_i^2\right) - (\text{correct mean})^2}$$

$$= \sqrt{\left(\frac{1}{19} \times 2016\right) - \left(\frac{192}{19}\right)^2} = \sqrt{\frac{1440}{361}}$$

$$= \frac{(12 \times 3.162)}{19} = 1.997$$

2. (a, c, d) : Given equation is

$$x^{\{(\log_3 x)^2 - (9/2)\log_3 x + 5\}} = 3\sqrt{3}$$

$$\Rightarrow \quad x^{\{(\log_3 x)^2 - (9/2)\log_3 x + 5\}} = 3^{3/2}$$

$$\Rightarrow \quad \{(\log_3 x)^2 - \left(\frac{9}{2}\right)\log_3 x + 5\}\log_3 x = \frac{3}{2} \qquad \ldots\text{(i)}$$

Put $\log_3 x = t$

Then from eqn (i), $t^3 - \frac{9}{2}t^2 + 5t = \frac{3}{2}$

$\Rightarrow \quad 2t^3 - 9t^2 + 10t - 3 = 0$
$\Rightarrow \quad (t-1)(2t^2 - 7t + 3) = 0$

$$\Rightarrow \quad (t-1)\left(t - \frac{1}{2}\right)(t-3) = 0$$

$$\therefore \quad t = \frac{1}{2}, 1, 3 \text{ or } \log_3 x = \frac{1}{2}, 1, 3 \Rightarrow x = \sqrt{3}, 3, 27$$

3. (c) : Given, $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in A.P.

$$\therefore \quad b - \frac{a+b}{1-ab} = \frac{b+c}{1-bc} - b \Rightarrow \frac{-a-ab^2}{1-ab} = \frac{c+b^2c}{1-bc}$$

$$\Rightarrow \quad (-a - ab^2)(1 - bc) = (1 - ab)(c + b^2c)$$

$$\Rightarrow \quad (1 + b^2)(a + c - 2abc) = 0$$

Since, $1 + b^2 \neq 0$

$$\therefore \quad a + c - 2abc = 0 \quad \Rightarrow \quad \frac{1}{b} = \frac{2ac}{a + c}$$

$\therefore \quad a, \frac{1}{b}, c$ are in H.P.

4. (a, b, d) : Let $z = \alpha$ be the real root of the given equation. Then

$$\alpha^3 + (3 + 2i)\alpha + (-1 + ia) = 0$$

$$\Rightarrow \quad (\alpha^3 + 3\alpha - 1) + i(a + 2\alpha) = 0$$

$$\Rightarrow \quad \alpha^3 + 3\alpha - 1 = 0 \text{ and } \alpha = -a/2$$

$$\Rightarrow \quad -\frac{a^3}{8} - \frac{3a}{2} - 1 = 0 \Rightarrow a^3 + 12a + 8 = 0$$

Let $f(a) = a^3 + 12a + 8$

$f(-1) < 0, f(0) > 0, f(-2) < 0, f(3) > 0, f(1) > 0$

$\Rightarrow \quad a \in (-2, 1)$ or $(-1, 0)$ or $(-2, 3)$

5. (a, c) : Consider $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

Let one of the vertices be $(a, 0)$.

Foci are $(\pm ae, 0)$

According to the question, we have

$ae - a = 1$ and $ae + a = 3$

Solving, we get $a = 1$ and $e = 2$

$$\therefore \quad e^2 = 1 + \frac{b^2}{a^2} \Rightarrow b^2 = 3$$

$\therefore$ Equation of hyperbola is $\frac{x^2}{1} - \frac{y^2}{3} = 1$

or $\quad 3x^2 - y^2 = 3$

For $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1, \quad e^2 = 1 + \frac{a^2}{b^2} \Rightarrow b^2 = \frac{1}{3}$

$\therefore$ Equation of hyperbola is $x^2 - 3y^2 + 1 = 0$

6. (c, d) : $\sin^{-1} x > \cos^{-1} x$

$$\Rightarrow \quad \sin^{-1} x > \frac{\pi}{2} - \sin^{-1} x$$

$$\Rightarrow \quad \sin^{-1} x > \frac{\pi}{4} \quad \Rightarrow \quad x > \sin\frac{\pi}{4}$$

$$\Rightarrow \quad x > \frac{1}{\sqrt{2}} \text{ and in } \sin^{-1} x,\ 0 < x < 1 \quad \therefore \quad x \in \left(\frac{1}{\sqrt{2}}, 1\right)$$

7. (2) : Since, $|z| = 2$ represents a circle with centre $(0, 0)$ and radius 2.

Let, O is the origin. O is also the centre of the circle $|z| = 2$.

Since, equilateral ΔABC is inscribed in the circle.

AVAILABLE BOUND VOLUMES

of your favourite magazines

2021

Physics For You (January - December)	₹ 380	12 issues
Chemistry Today (January - December)	₹ 380	12 issues
Mathematics Today (January - December)	₹ 380	12 issues
Biology Today (January - December)	₹ 380	12 issues

2019

Mathematics Today (January - December)	₹ 380	12 issues

2018

Physics For You (January - December)	₹ 380	12 issues
Chemistry Today (January - December)	₹ 380	12 issues
Mathematics Today (January - December)	₹ 380	12 issues
Biology Today (January - December)	₹ 380	12 issues

How to order : Send money by demand draft/money order. Demand Draft should be drawn in favour of **MTG Learning Media (P) Ltd.** Mention the volume you require along with you name and address. OR buy online from www.mtg.in

Add ₹ 90 as postal charges

Older issues can be accessed on **digital.mtg.in** in digital form.

Mail your order to :

Circulation Manager,
MTG Learning Media (P) Ltd.
Plot No. 99, Sector 44
Institutional Area, Gurugram, (HR)
Tel.: (0124) 6601200
E-mail : info@mtg.in
Web : www.mtg.in

buy online at www.mtg.in

Hence, O is the circumcentre of ΔABC.

$\therefore \quad \angle AOB = \angle BOC = \angle COA = 120^\circ = \frac{2\pi}{3}$

Now, $z_1 = 2e^{\frac{i\pi}{3}}$; $z_2 = z_1 e^{\frac{2\pi i}{3}} = 2e^{i\pi}$ $\therefore z_2 = -2$

8. (1) :

$$z_3 = z_2 e^{\frac{2\pi i}{3}} = (-2)\left(\frac{-1}{2}+\frac{i\sqrt{3}}{2}\right) = 1-\sqrt{3}i \ \therefore\ z_3 = 1-\sqrt{3}i$$

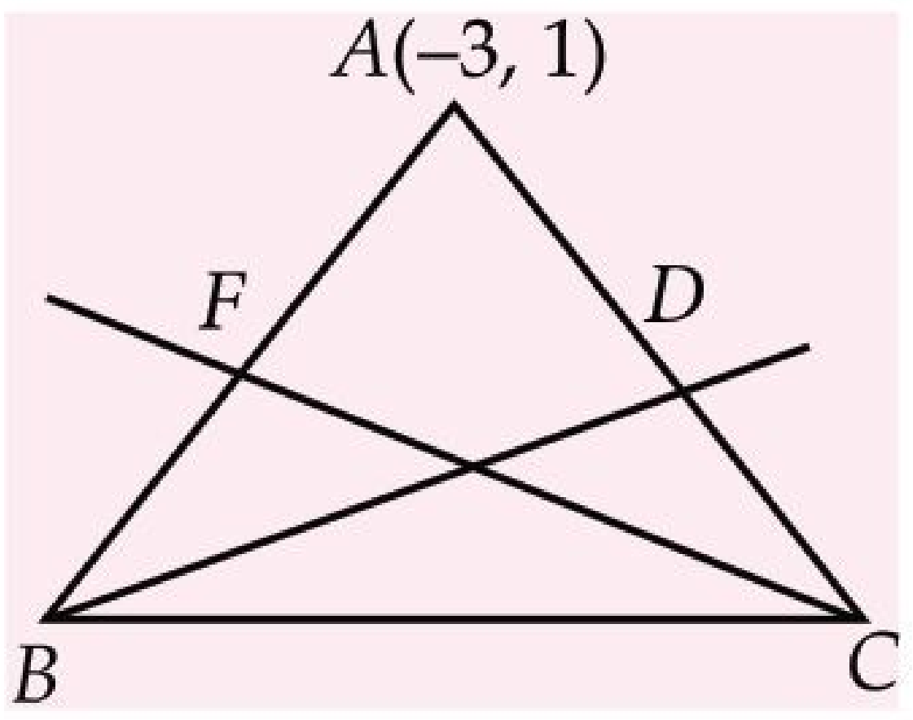
9. (9) : $BD : 2x + y = 3$ is the median and $CF : 7x - 4y = 1$ is the angular bisector

$D = (\alpha, 3 - 2\alpha)$ is the midpoint of AC.

$\therefore \quad C = (2\alpha + 3, 5 - 4\alpha)$ is a point on $7x - 4y = 1$.

$\therefore \quad 7(2\alpha + 3) - 4(5 - 4\alpha) = 1$

$\Rightarrow \quad \alpha = 0$ and $C = (3, 5)$



Equation of AC is,

$$\frac{y-1}{x+3} = \frac{5-1}{3+3} = \frac{2}{3}$$

$\Rightarrow \quad 2x - 3y + 9 = 0$

$\therefore \quad$ The distance of origin *i.e.*, (0, 0) from AC is $\frac{9}{\sqrt{13}}$.

10. (49) : Slopes of CA, CF, CB are $\frac{2}{3}, \frac{7}{4}, m$ respectively and CF bisects the angle between CAF and CBF.

So, $$\frac{m-\frac{7}{4}}{1+\frac{7m}{4}} = \frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{14}{12}} = \frac{1}{2}$$

$\therefore \quad 2m - \frac{7}{2} = 1 + \frac{7m}{4} \Rightarrow m = 18$

$\therefore \quad$ Equation of BC is $\frac{y-5}{x-3} = 18 \ \Rightarrow\ 18x - y = 49$

$\therefore \quad$ The distance of origin *i.e.*, (0, 0) from BC is $\frac{49}{\sqrt{18^2+1}} = \frac{49}{5\sqrt{13}}$.

(11-12) :

$$\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} a & y & z \\ x-a & b-y & 0 \\ x-a & 0 & c-z \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \frac{a}{x-a} & \frac{y}{y-b} & \frac{z}{z-c} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{a}{x-a} - \frac{y}{y-b}(-1) + \frac{z}{z-c}(1) = 0$$

$$\Rightarrow \left(\frac{x}{x-a} - 1\right) + \frac{y}{y-b} + \frac{z}{z-c} = 0$$

$$\Rightarrow \frac{x}{x-a} + \frac{y}{y-b} + \frac{z}{z-c} = 1$$

$$\Rightarrow \frac{2x}{x-a} + \frac{2y}{y-b} + \frac{2z}{z-c} = 2$$

$$\Rightarrow \frac{x+a}{x-a} + 1 + \frac{y+b}{y-b} + 1 + \frac{z+c}{z-c} + 1 = 2$$

$$\Rightarrow \frac{x+a}{x-a} + \frac{y+b}{y-b} + \frac{z+c}{z-c} = -1$$

11. (1) **12. (1)**

13. (b) : Let $b + c = x$, $bc = y$

Since, A.M. ≥ G.M.

$\therefore \ \frac{b+c}{2} \ge \sqrt{bc}$

$\Rightarrow \ x \ge 2\sqrt{y}$ and $z = a$

Then $(b + c + a)(b + c - a) = bc$

$[\because \ (x + z)(x - z) = y]$

$\Rightarrow \quad 2s(2s - 2a) = bc$

$$\Rightarrow \sqrt{\frac{s(s-a)}{bc}} = \frac{1}{2} \ \Rightarrow\ \cos\left(\frac{A}{2}\right) = \cos\left(\frac{\pi}{3}\right)$$

$\therefore \ A = \frac{2\pi}{3} = 120^\circ$

14. (d) : $R = \frac{a}{2\sin A} = \frac{z}{2\sin 120^\circ} = \frac{z}{\sqrt{3}}$

15. (c) : $r_3 = s\tan\frac{C}{2} = s = \frac{\Delta}{r}$

$\Delta^2 = r_1 r_2 r_3 r = r_1 r_2 \Delta$

$\therefore \quad \Delta = r_1 r_2 = 39 \times 70 = 3 \cdot 13 \cdot 2 \cdot 5 \cdot 7.$

16. (a) : $r_1 + r_2 = 4R \cdot \cos^2\frac{C}{2} = 2R = c$

$\therefore \quad c = 39 + 70 = 109.$

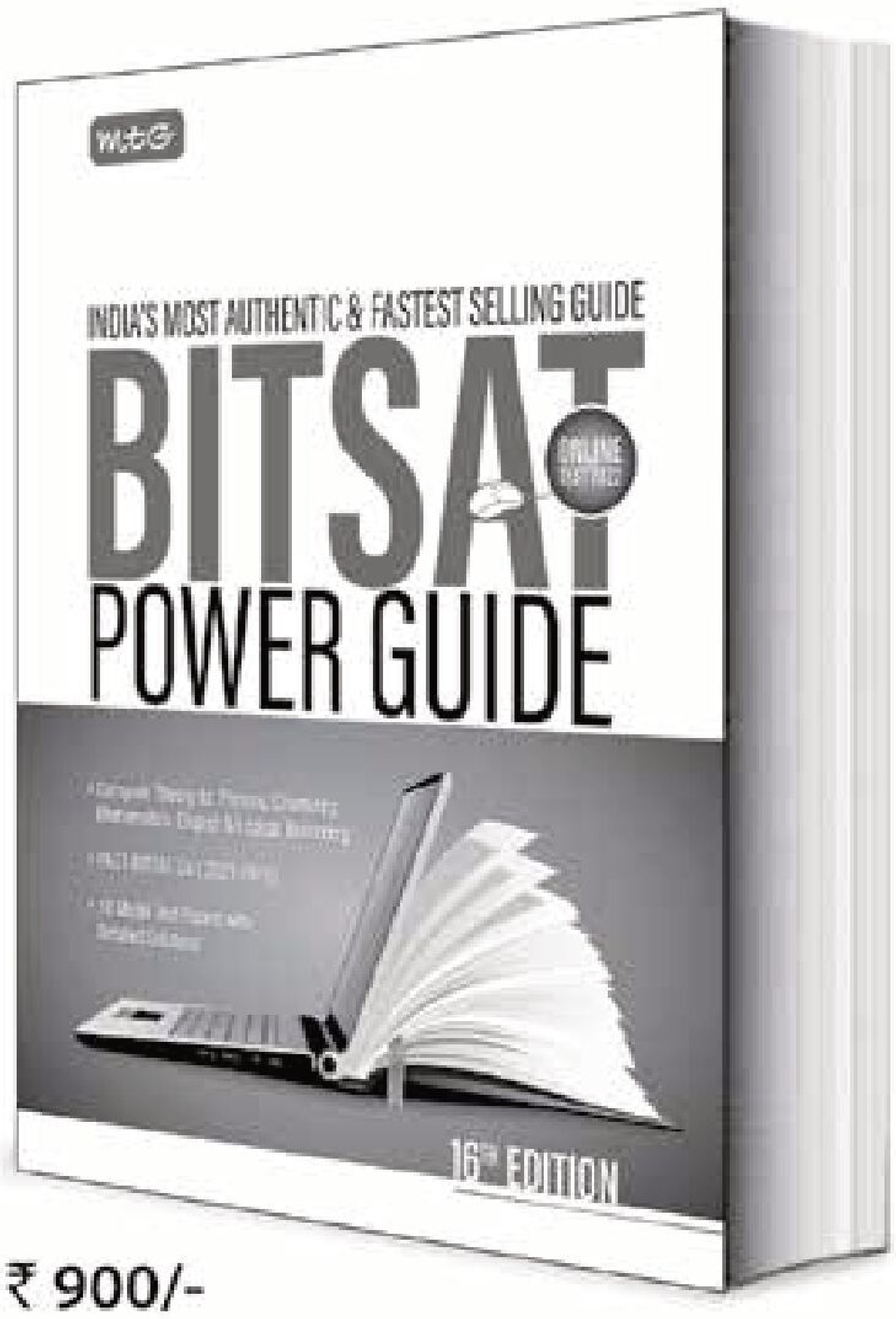
17. (4410) **18. (1)** **19. (1005)**

◆◆

mtG

FULLY LOADED & COMPLETELY UPDATED

MTG's BITSAT Power Guide is not only the most exhaustive prep-tool, but also the only book available at present, updated as per the latest BITSAT syllabus for students aspiring for top rank in BITSAT 2022.



₹ 900/-

Get MTG's BITSAT Power Guide today for a real-world feel of BITSAT. Find out what's different about the BITSAT test, including its pattern of examination and key success factors. Be it with chapterwise MCQs or model test papers, check how good your chances are for glory in BITSAT 2022.

FEATURES

- Covers all 5 subjects - Physics, Chemistry, Mathematics, English & Logical Reasoning
- Chapterwise MCQs in each section for practice
- Past chapterwise BITSAT Qs (2021-2016)
- 10 Model Test Papers with detailed solutions

Visit **www.MTG.in** to buy online. Or visit a leading bookseller near you.
For more information, email *info@mtg.in* or call **1800-10-38673** (toll-free).

CONCEPT MAP

STATISTICS

Class XI

Measures of Central Tendency

Mean, Median, Mode

Analysis of Frequency Distributions

Coefficient of variation (C.V.) $= \frac{\text{S.D.}}{\text{Mean}} \times 100 = \frac{\sigma}{\bar{x}} \times 100,\ \bar{x} \neq 0$

- The data having higher C.V. will be more variable and the data having lower C.V. will be more consistent.

For Ungrouped Data

Let $x_1, x_2, \ldots\ldots, x_n$ be n observations, then mean deviation

- About mean $= \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$, where $\bar{x}$ = Mean
- About median $= \frac{1}{n}\sum_{i=1}^{n}|x_i - M|$, where M = Median

For Grouped Data

Let $x_1, x_2, \ldots, x_n$ be a set of n observations occurring with frequencies $f_1, f_2, \ldots, f_n$ respectively, then mean deviation

- About mean $= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$, where $\bar{x}$ = mean
- About median $= \frac{1}{N}\sum_{i=1}^{n} f_i\,|x_i - M|$, where M = Median

Here, x_i are the mid-points of classes and $N = \sum_{i=1}^{n} f_i$ = Sum of all frequencies.

Short Cut Method

- Mean deviation about mean $= \frac{1}{N}\sum_{i=1}^{n} f_i|x_i - \bar{x}|$,

  where mean, $(\bar{x}) = a + \frac{\sum_{i=1}^{n} f_i u_i}{N} \times h$, where $u_i = \frac{x_i - a}{h}$, a is assumed mean and h is width of class-interval.
- Mean deviation about median $= \frac{1}{N}\sum_{i=1}^{n} f_i\,|x_i - M|$, where

  median, $(M) = l + \frac{\frac{N}{2} - cf}{f} \times h$, where l, f and cf are lower limit, frequency and cumulative frequency.

Measures of Dispersion

Range

Modulus of the difference between two extreme observations of the given distribution.

Range = Maximum Value – Minimum Value

Quartiles

Quartiles are those values of the variate which divide the distribution into four equal parts.

Mean Deviation

It is the arithmetic mean of the absolute values of deviations about some point (mean or median or mode).

Mean Deviation $= \frac{\text{Sum of Deviations}}{\text{Number of Observations}}$

Standard Deviation

Positive square root of the variance is known as standard deviation and it is denoted by σ.

For ungrouped data	$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$
For grouped data	$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i(x_i - \bar{x})^2}$
Shortcut method	$\sigma = \frac{h}{N}\left[\sqrt{N\sum_{i=1}^{n} f_i u_i^2 - \left(\sum_{i=1}^{n} f_i u_i\right)^2}\right]$ where $u_i = \frac{x_i - a}{h}$, a = assumed mean, h = width of class-intervals

Variance of Combined Series

Let $\bar{x}_1$ and $\bar{x}_2$ be means of two groups with standard deviations σ_1, σ_2 and number of elements n_1 and n_2, then variance is given by

$$(\sigma^2) = (\text{S.D.})^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

where $d_1 = \bar{x}_1 - \bar{x}$, $d_2 = \bar{x}_2 - \bar{x}$ and $\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

INDEFINITE INTEGRALS

Class XII

CONCEPT MAP

Integration is the inverse process of differentiation $\frac{d}{dx}F(x) = f(x)$, $F(x)$ is an indefinite integral of $f(x)$ or, $\int f(x)dx = F(x) + c$, where c is an arbitrary constant of integration

Properties

- $\int f'(x)dx = f(x) + c$
- $\int[f(x) \pm g(x)]dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\int k \cdot f(x)\,dx = k\int f(x)\,dx$, k being any real number.

INDEFINITE INTEGRALS

Methods

Using Substitution

The given integral $\int f(x)\,dx$ can be transformed into another form by changing the independent variable x to t by substituting $x = g(t)$.

Using by Parts

If u and v are two differentiable functions of x, then

$$\int(uv)\,dx = \left[u \cdot \int v dx\right] - \int\left\{\frac{du}{dx} \cdot \int v dx\right\}dx.$$

In order to choose 1st function, we take the letter which comes first in the word ILATE.

I – Inverse Trigonometric Function,

L – Logarithmic Function, A – Algebraic Function,

T – Trigonometric Function, E – Exponential Function

Using Partial Fractions

- If $f(x)$ and $g(x)$ are two polynomials such that $\deg f(x) \geq \deg g(x)$, then we divide $f(x)$ by $g(x)$.

  $\therefore\ \frac{f(x)}{g(x)} = \text{Quotient} + \frac{\text{Remainder}}{g(x)}$
- If $f(x)$ and $g(x)$ are two polynomials such that the degree of $f(x)$ is less than the degree of $g(x)$, then we can evaluate $\int\frac{f(x)}{g(x)}dx$ by decomposing $\frac{f(x)}{g(x)}$ into partial fraction.

Some Standard Integrals

- $\int dx = x + c$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, where $n \neq -1$
- $\int e^x dx = e^x + c$
- $\int a^x dx = \frac{a^x}{\log_e a} + c$, where $a > 0, a \neq 1$
- $\int \frac{1}{x}dx = \log_e|x| + c$, where $x \neq 0$
- $\int \sin x\,dx = -\cos x + c$
- $\int \cos x\,dx = \sin x + c$
- $\int \tan x\,dx = \log|\sec x| + c$
- $\int \cot x\,dx = \log|\sin x| + c$
- $\int \sec x\,dx = \log|\sec x + \tan x| + c$
- $\int \operatorname{cosec} x\,dx = \log|\operatorname{cosec} x - \cot x| + c$

where 'c' is the constant of integration.

Integrals of Some Particular Functions

- $\int\frac{1}{\sqrt{x^2 - a^2}}dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int\frac{1}{\sqrt{x^2 + a^2}}dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int\frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c$
- $\int\frac{1}{a^2 - x^2}dx = \frac{1}{2a}\log\left|\frac{a + x}{a - x}\right| + c$
- $\int\frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$
- $\int\frac{1}{x\sqrt{x^2 - 1}}dx = \sec^{-1}x + c = -\operatorname{cosec}^{-1}x + c$, where $|x| > 1$
- $\int\sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$
- $\int\sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$
- $\int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int\frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
- $\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$

beat the TIME TRAP

Duration : 30 minutes

SECTION-I

Only One Option Correct Type

1. Let $2(1+x^3)^{100} = \sum_{i=0}^{100}\left(a_i x^i - \cos\frac{\pi(x+i)}{2}\right)$ then $\sum_{i=0}^{50} a_{2i} =$
   (a) 2^{100} (b) 2^{99} (c) 2^{50} (d) 2^{49}

2. From any point on the circle $x^2 + y^2 = 9$, two tangents are drawn to circle $x^2 + y^2 = 4$ which cut $x^2 + y^2 = 9$ at A and B then locus of point of intersection of tangents drawn to $x^2 + y^2 = 9$ at A and B is
   (a) $x^2 + y^2 = 27$ (b) $x^2 + y = 27$
   (c) $x^2 + y^2 = 27^2$ (d) $x^2 + y = 27^2$

3. Let A be a 2×2 matrix with real entries and $\det(A) = t \neq 0$ such that $\det(A + t(\operatorname{adj} A)) = 0$ then $\det(A - t(\operatorname{adj} A)) =$
   (a) 0 (b) 2 (c) 4 (d) 8

4. Let $A(\omega^2)$, $B(2i\omega)$, $C(-4)$ be three points lying on the complex plane. On the circumcircle of ΔABC, a point P is taken such that $PA \cdot BC = PC \cdot AB$ (where P, A, B, C are in order). If z is the affix of midpoint of PB then $|z|^2 =$
   (a) 0 (b) 1 (c) 2 (d) 3

5. Let 'a' and 'b' be distinct, randomly chosen roots of the equation $z^{210} - 1 = 0$. The probability that $|a+b| \geq \sqrt{2+\sqrt{3}}$ is (approx.)
   (a) 0.16 (b) 0.22 (c) 0.31 (d) 0.54

6. A bag contains 21 red and 21 black balls. We remove 2 balls at a time repeatedly and discard them if they are of the same colour, but if they are different then we discard the black ball and return the red ball. The probability that this process will terminate with 1 red ball in the bag is
   (a) 0 (b) 1/2 (c) 1/3 (d) 1

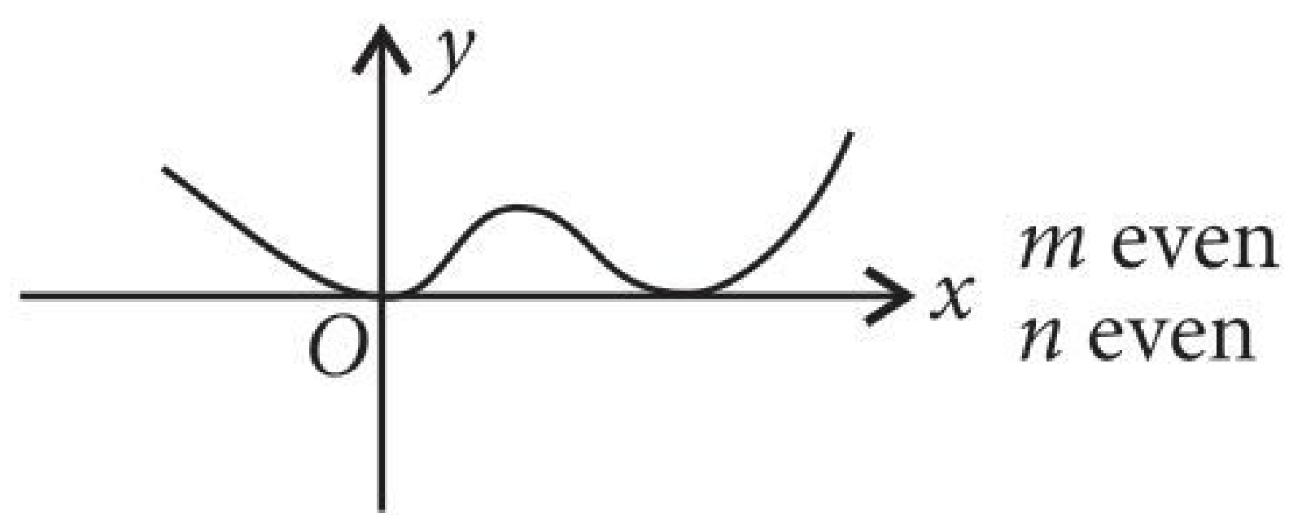
7. $\int_0^{\infty} (\sqrt{x^2+1}+x)^{-3}\, dx =$
   (a) 1/8 (b) 2/8 (c) 3/8 (d) 4/8

SECTION-II

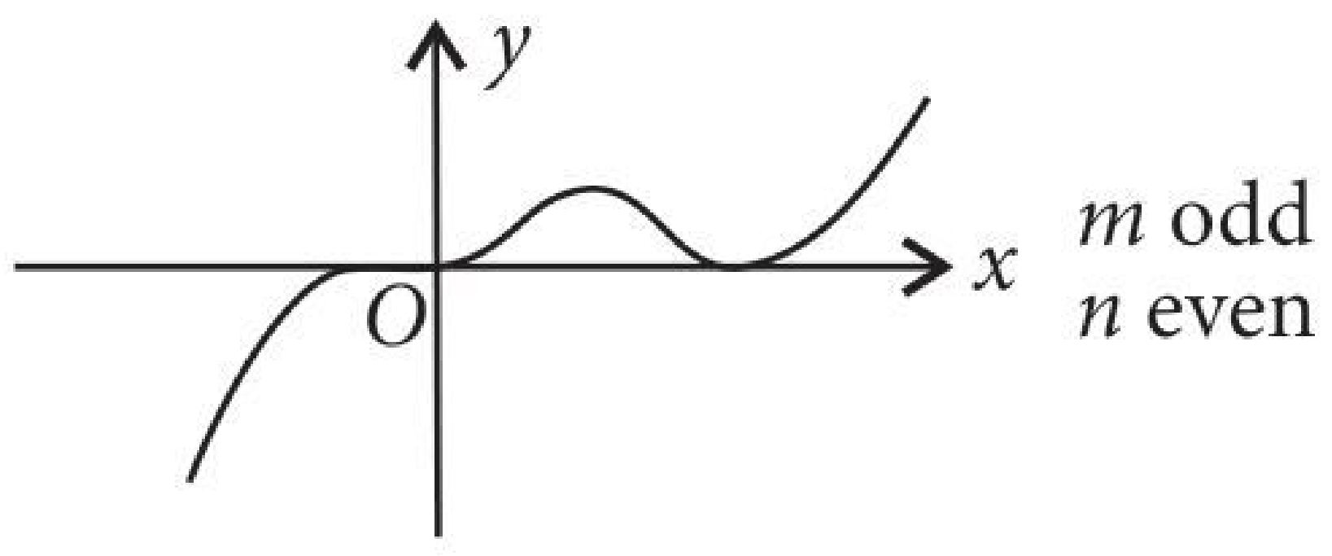
One or More than One Option(s) Correct Type

8. Let $f(x) = x^m(1 - x)^n$, $m, n \in I$ and $(m, n > 1)$ then possible graph of $f(x)$ in various cases of m and n is/are

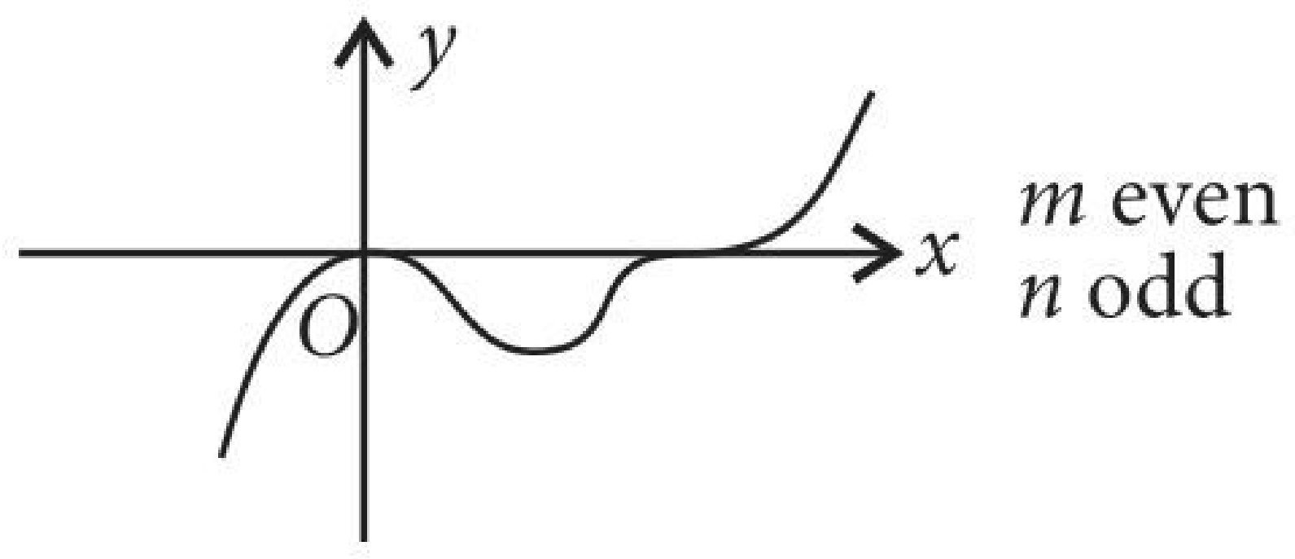
(a)



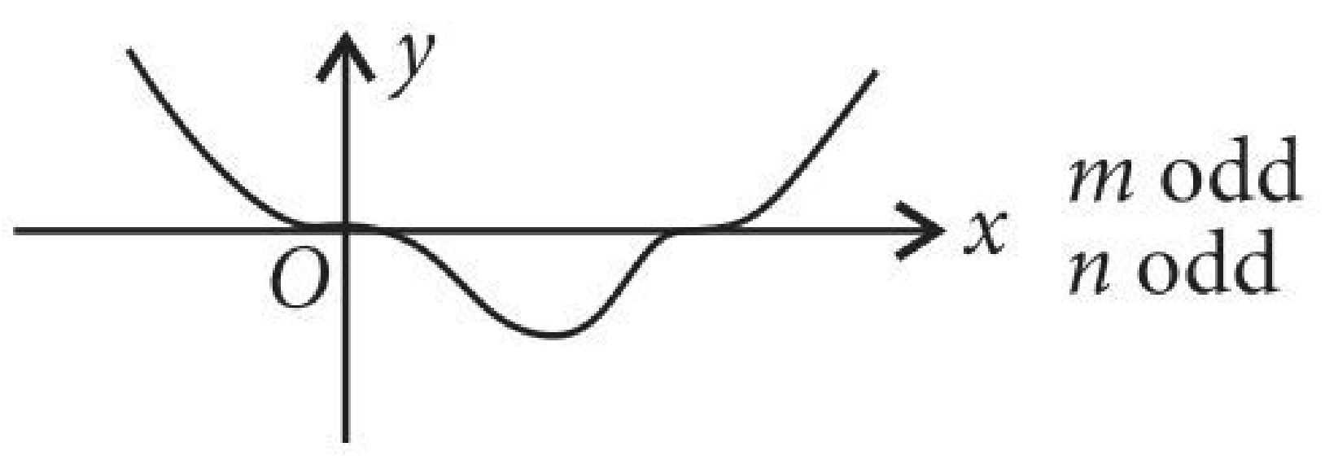
(b)



(c)

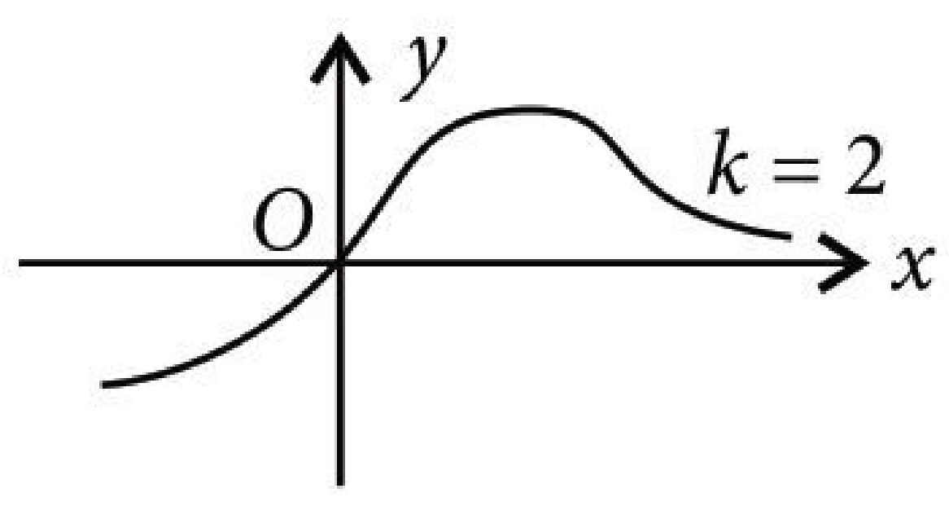


(d)

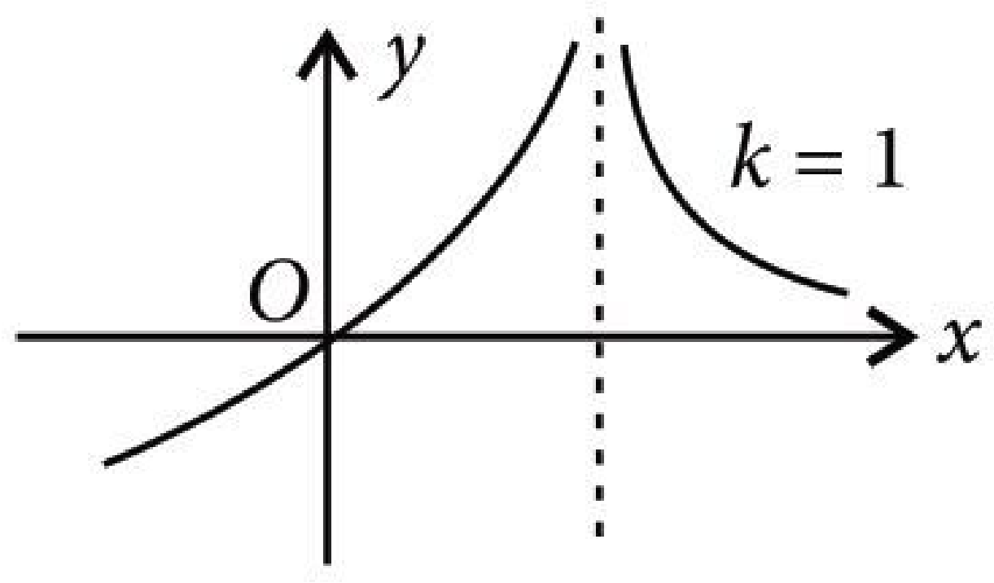


9. The possible graph of $f(x) = \dfrac{x}{\sqrt{x^2 - 2x + k}}$ is/are

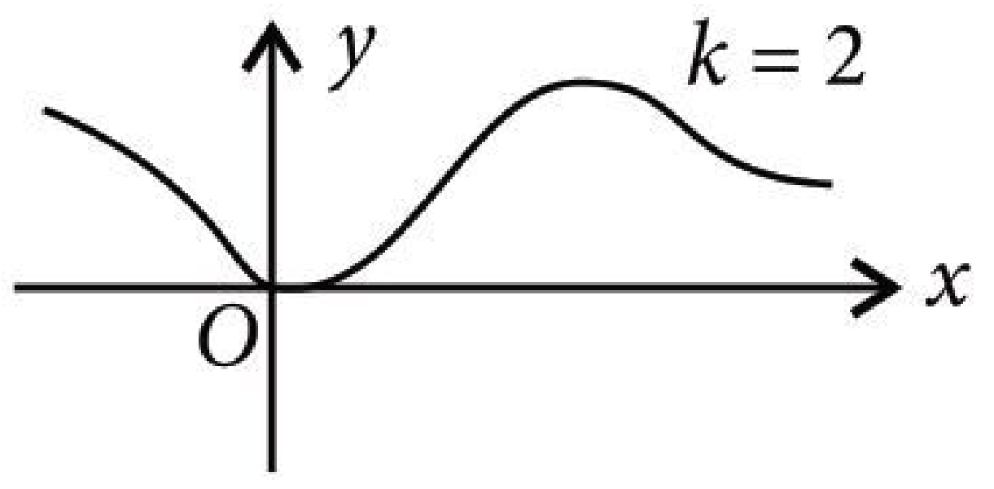
(a)



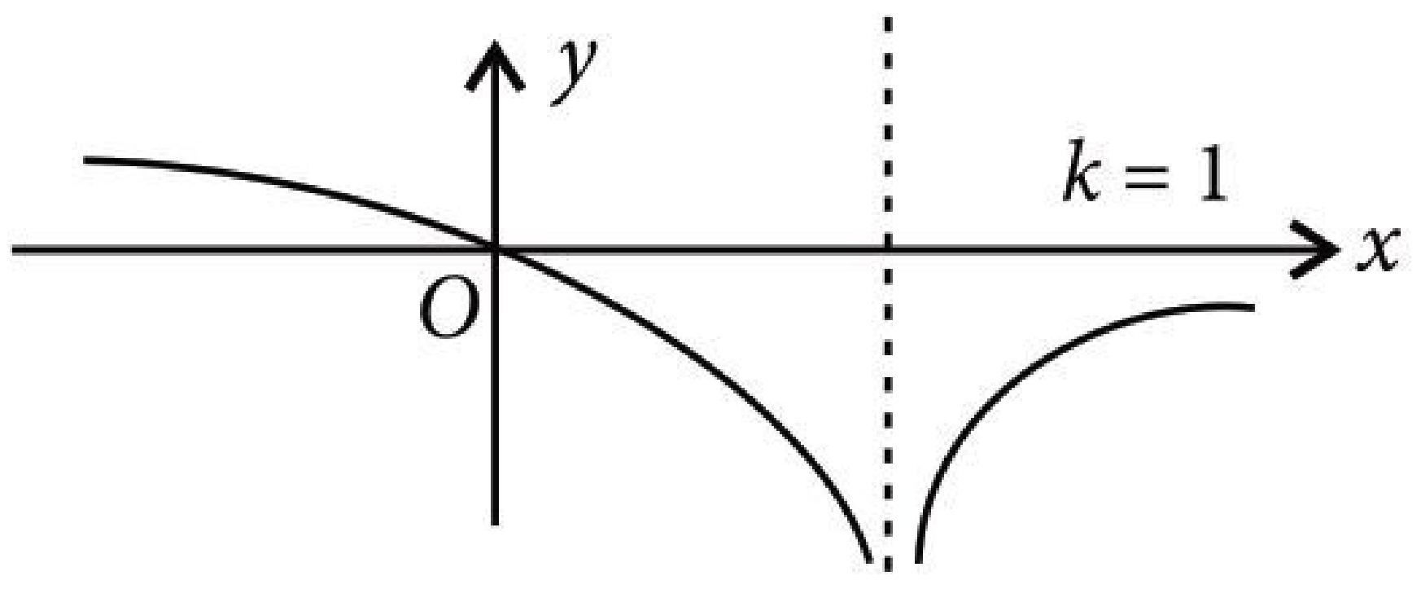
(b)



(c)



(d)



10. A damped system with feedback is given by the differential equation $f'(t) = -f(t) + kf(t-1)$ where k is a given non-zero constant. The non-zero solutions of f of the form $f(t) = ae^{mt}$, where 'a' and 'm' are constants, are possible provided m satisfies the equation $g(m) = 0$. Then

(a) $g(m) = (m+1) - ke^{-m}$

(b) $g(m) = (m-1) + ke^{m}$

(c) for $k > 0$, $g(m) = 0$ has no solution in m

(d) for $k > 0$, $g(m) = 0$ has exactly one solution in m

11. The possible value of the expression

$$\frac{\sec^4\alpha}{\tan^2\beta}+\frac{\sec^4\beta}{\tan^2\alpha}\quad\left(\alpha,\beta\neq\frac{n\pi}{2},\ n\in I\right)\text{ is/are}$$

(a) 6 (b) 8 (c) 10 (d) 12

12. Let $\theta = \pi/7$ then

(a) $\cos\theta$ is not a root of the equation $8x^3 + 4x^2 - 4x - 1 = 0$

(b) $\cos\theta$ is irrational

(c) $\sin 4\theta = \sin 3\theta$

(d) $\sin^2 3\theta - \sin^2\theta = \sin 2\theta \sin 3\theta$

13. A straight line through $P(\lambda, 2)$, $(\lambda \neq 0)$ meets the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A and D meets the axes at B and C such that PA, PB, PC, PD are in G.P. then $\lambda \in$

(a) $[7, \infty)$ (b) $(-12, -8)$

(c) $(-5, 0)$ (d) $(10, \infty)$

14. Let $P(x) = x^3 + x + 1$ and $Q(x)$ be a polynomial such that the roots of $Q(x) = 0$ are the squares of the roots of $P(x) = 0$ then

(a) $Q(4)$ is divisible by 11

(b) the last two digits of $(Q(1))^{50}$ is 29

(c) when $(Q(1))^{50}$ is divided by 5, then remainder is 4

(d) the probability that $Q(n)$ is an odd integer is 1; $(n \in N)$

SECTION-III

Comprehension Type

Passage-1

In a triangle ABC, if $r_1 = 1$, $r_2 = 2$ and $r_3 = 3$, then

15. The perimeter of the triangle is

(a) $\sqrt{22}$ (b) $\sqrt{33}$

(c) $\sqrt{44}$ (d) $\sqrt{55}$

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

3				8								9				7			2	
7	8					4	1							1		2				
		1			7		3	9							1				9	4
		5	6	9			2							5		8				
							5						4				6			9
		8		5		3						8					5	4		7
1		2	7	3	8					8	1					1				
						5						2					7	5		6
		3				1			9	3				6						
									8		9	7								
							3		1	6										
						6					2		9							
		7		9	8	2						8	4							
8								4						5			7	8		
2	5			1		3			5					2	8		9			1
					9	8							1	4	6				8	3
5		3					2													
					6		5								3				9	5
	8					4	7						2			7		4		
		1	4			5							9				4	5		
			6			9							5	8		2	3			

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

16. The area of the triangle, is

(a) $\frac{2}{\sqrt{11}}$ (b) $\frac{3}{\sqrt{11}}$

(c) $\frac{6}{\sqrt{11}}$ (d) $\frac{3}{\sqrt{7}}$

Passage-2

For certain curve $y = f(x)$ satisfying $\frac{d^2y}{dx^2} = 6x - 4, f(x)$ has local minimum value 5 when $x = 1$.

17. Number of critical points for $y = f(x)$ for $x \in (0, 1)$ are

(a) 0 (b) 1 (c) 2 (d) 3

18. Global maximum value of $y = f(x)$ for $x \in [0, 2]$ is

(a) 5 (b) 7 (c) 8 (d) 9

SOLUTIONS

1. (a) : Given $2(1 + x^3)^{100} = (a_0 + a_1x + ... + a_{100}x^{100})$

$$-\left(\cos\frac{\pi x}{2} + \cos\frac{\pi(x+1)}{2} + ... + \cos\frac{\pi}{2}(x+100)\right)$$

Putting $x = 1$ and $x = -1$ alternately and then adding, we have $\sum_{i=0}^{50} a_{2i} = 2^{100}$

2. (c) : Let $R(h, k)$ be the point of intersection of the tangents at A and B then from ΔOAP, $\sin\theta = 2/3$ where $\angle OAP = \theta$ and so $\angle AOR = \pi - 2\theta$ gives

$$\cos(\pi - 2\theta) = \frac{3}{OR} = \frac{3}{\sqrt{h^2 + k^2}}.$$

$$\Rightarrow \quad -\cos 2\theta = \frac{3}{\sqrt{h^2 + k^2}} \Rightarrow -(1 - 2\sin^2\theta) = \frac{3}{\sqrt{h^2 + k^2}}$$

$$\Rightarrow \quad -\left(1 - 2 \times \frac{4}{9}\right) = \frac{3}{\sqrt{h^2 + k^2}} \Rightarrow \sqrt{h^2 + k^2} = 27$$

Hence, locus is circle $x^2 + y^2 = (27)^2$.

3. (c) : Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $ad - bc = t$ and

$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ and

$$|A + t \text{ adj}(A)| = 0 \Rightarrow \begin{vmatrix} a + td & b - bt \\ c - ct & d + at \end{vmatrix} = 0$$

On simplification, we get $(a + d)^2 + (t - 1)^2 = 0$

i.e., $a + d = 0$ and $t = 1$

and $|A - t(\text{adj}.A)| = t(1 + t)^2 = 4$

4. (a) : Let z_p be the affix of point P then applying Rotation theorem at P and B, we get

$$\frac{z_p + 4}{z_p - \omega^2} \cdot \frac{2i\omega - \omega^2}{2i\omega + 4} = -1$$

Simplifying, we have $z_p = -2i\omega$

So, $z = \frac{z_p + 2i\omega}{2} = 0$ *i.e.*, $|z|^2 = 0$

5. (a) : As all the 210 roots of the equation are symmetrically distributed in the complex plane, we can assume $a = 1$ without loosing generality. So, the given condition becomes

$|1 + b|^2 = |1 + \cos\theta + i\sin\theta|^2 = 2 + 2\cos\theta \geq 2 + \sqrt{3}$

(ATQ)

i.e. $\cos\theta \geq \frac{\sqrt{3}}{2}$ *i.e.*, $|\theta| \leq \frac{\pi}{6}$

As $b \neq 1$, θ is of the form $\pm\frac{2k\pi}{210}$, where

$$k \in \left[1, \left[\frac{210}{12}\right]\right] = [1, 17]$$

There are $2 \times 17 = 34$ such angles.

So, the required probability $= \frac{34}{209} \approx 0.16$

6. (d) : As atleast one ball is removed from each stage, the process will eventually end with either no ball or one ball. Because red balls are odd in number at start and we are removing 2 at a time, the number of red balls at any time is odd. Hence, the process will always leave red balls in the bag. So, it ends with exactly 1 red ball. Hence, required probability = 1.

7. (c) : Let $I = \int_0^\infty (\sqrt{x^2 + 1} + x)^{-3} dx$

Put $x + \sqrt{x^2 + 1} = t$ *i.e.*, $x = \frac{t^2 - 1}{2t}$

$$\therefore \quad I = \frac{1}{2}\int_1^\infty (t^{-3} + t^{-5})\, dt = \frac{3}{8}$$

8. (a, b, c, d) : $f'(x) = f(x) \cdot \left(\frac{m}{x} - \frac{n}{1 - x}\right)$

and $f''(x) = f(x)\left(\frac{-m}{x^2} - \frac{n}{(x-1)^2}\right) + f(x)\left(\frac{m}{x} + \frac{n}{x - 1}\right)^2$

If $f(x) > 0$ then $f''(x) < 0$ and if $f(x) < 0$, then $f''(x) > 0$.

9. (a, b) : Let $y = \dfrac{x}{\sqrt{x^2 - 2x + k}}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{(k - x)}{(x^2 - 2x + k)^{3/2}}$

When $k = 2$, the stationary point is at $(2, \sqrt{2})$ and $x = 0, y = 0$; $x \to \infty, y \to 1$; $x \to -\infty, y \to -1$

When $k = 1$, then $y = \dfrac{x}{|x - 1|}$ *i.e.*, vertical asymptote is at $x = 1$.

10. (a, d) : Putting $f(t) = ae^{mt}$ in the given differential equation, we have

$m\cdot e^{mt} = -e^{mt} + k\cdot e^{m(t-1)}$

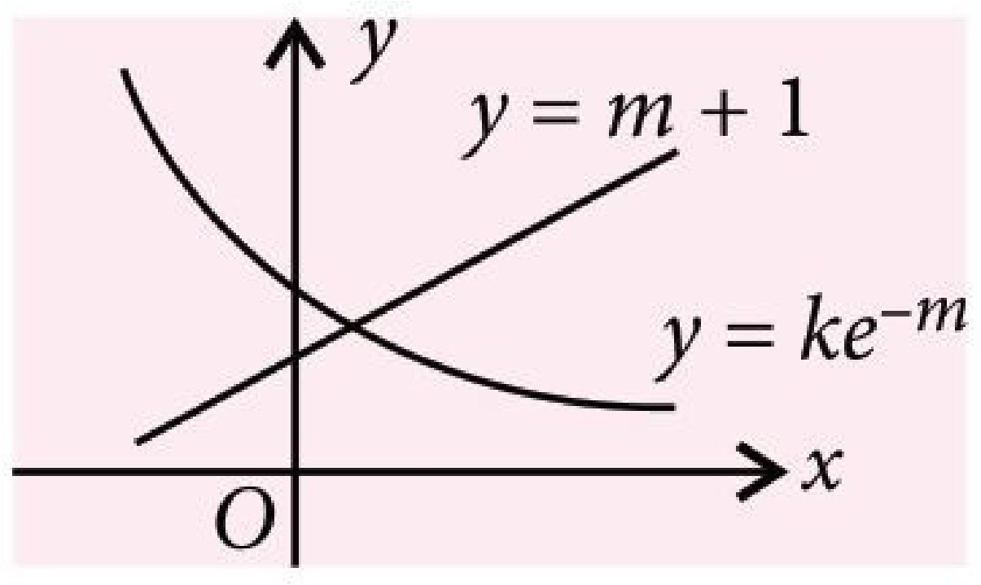
i.e. $m + 1 = ke^{-m}$

Hence, $g(m) = (m + 1) - ke^{-m}$

When $k > 0$, $g(m) = 0$

i.e., $(m + 1) = ke^{-m}$

Graphs intersect at only one point.



11. (b, c, d) : Let $\tan^2\alpha = a$, $\tan^2\beta = b$, then the given expression reduces to

$$\frac{(a+1)^2}{b} + \frac{(b+1)^2}{a} = 2\left(\frac{a}{b} + \frac{b}{a}\right) + \left(\frac{a^2}{b} + \frac{b^2}{a} + \frac{1}{a} + \frac{1}{b}\right)$$

$$\geq 2(2) + 4(1) = 8$$

Hence, minimum value $= 8$ at $a = b = 1$

i.e. $\alpha = n\pi \pm \dfrac{\pi}{4}$, $\beta = n\pi \pm \dfrac{\pi}{4}$

12. (a, b, c, d) : Let $\theta = \dfrac{\pi}{7}$, $7\theta = \pi \to 4\theta = \pi - 3\theta$

So, $\sin 4\theta = \sin 3\theta$

i.e., $2\sin 2\theta \cos 2\theta = 3\sin\theta - 4\sin^3\theta$

Simplifying further, we get $8\cos^3\theta - 4\cos^2\theta - 4\cos\theta + 1 = 0$

So, $(2\cos\theta)$ is root of the equation $t^3 - t^2 - 2t + 1 = 0$

Now, we see that $t = 1$ or $t = -1$ are not the roots.

So, $(2\cos\theta)$ is irrational $\Rightarrow$ $\cos\theta$ is irrational.

13. (a, b, d) : Any point on the conic through $P(\lambda, 2)$ is $(\lambda + r\cos\theta, 2 + r\sin\theta)$ and meets the ellipse at A and D

So, $\dfrac{(\lambda + r\cos\theta)^2}{9} + \dfrac{(2 + r\sin\theta)^2}{4} = 1$

So, $r_1 r_2 = PA \cdot PD = \dfrac{4\lambda^2}{4\cos^2\theta + 9\sin^2\theta}$

Again solving with $x = 0$ and $y = 0$, we have

$PB \cdot PC = \dfrac{2\lambda}{\sin\theta\cos\theta}$

As PA, PB, PC, PD are in G.P.

$\Rightarrow \dfrac{4\lambda^2}{4\cos^2\theta + 9\sin^2\theta} = \dfrac{2\lambda}{\sin\theta\cos\theta}$

i.e., $2\lambda\sin 2\theta + 5\cos 2\theta = 13$

$\Rightarrow$ $18\tan^2\theta - 4\lambda\tan\theta + 8 = 0$

Now, $D > 0$

$\Rightarrow$ $16\lambda^2 - 4 \times 8 \times 18 > 0 \Rightarrow \lambda^2 > 36 \Rightarrow \lambda > 6$

14. (a, c, d) : Let $x = \alpha^2$, $\alpha = \sqrt{x}$ put in $P(x) = 0$, we get

$x\sqrt{x} + \sqrt{x} + 1 = 0$

$\Rightarrow$ $\sqrt{x}(x + 1) = -1$

$\Rightarrow$ $x^3 + 2x^2 + x - 1 = 0$

$\Rightarrow$ $\theta(x) = x^3 + 2x^2 + x - 1$

(15-16):

15. (c) : $2s = 2\sqrt{r_1r_2 + r_2r_3 + r_3r_1} = 2\sqrt{11} = \sqrt{44}$

16. (c) : $\Delta = \dfrac{r_1\, r_2\, r_3}{s} = \dfrac{6}{\sqrt{11}}$

(17-18):

17. (b) 18. (b)

$\dfrac{d^2y}{dx^2} = 6x - 4 \Rightarrow \dfrac{dy}{dx} = 3x^2 - 4x + c$

local min. at $x = 1 \Rightarrow \left(\dfrac{dy}{dx}\right)_{x=1} = 0 \Rightarrow c = 1$

$\therefore \dfrac{dy}{dx} = 3x^2 - 4x + 1 \Rightarrow y = x^3 - 2x^2 + x + d$

When $x = 1$, $y = 5 \Rightarrow d = 5$

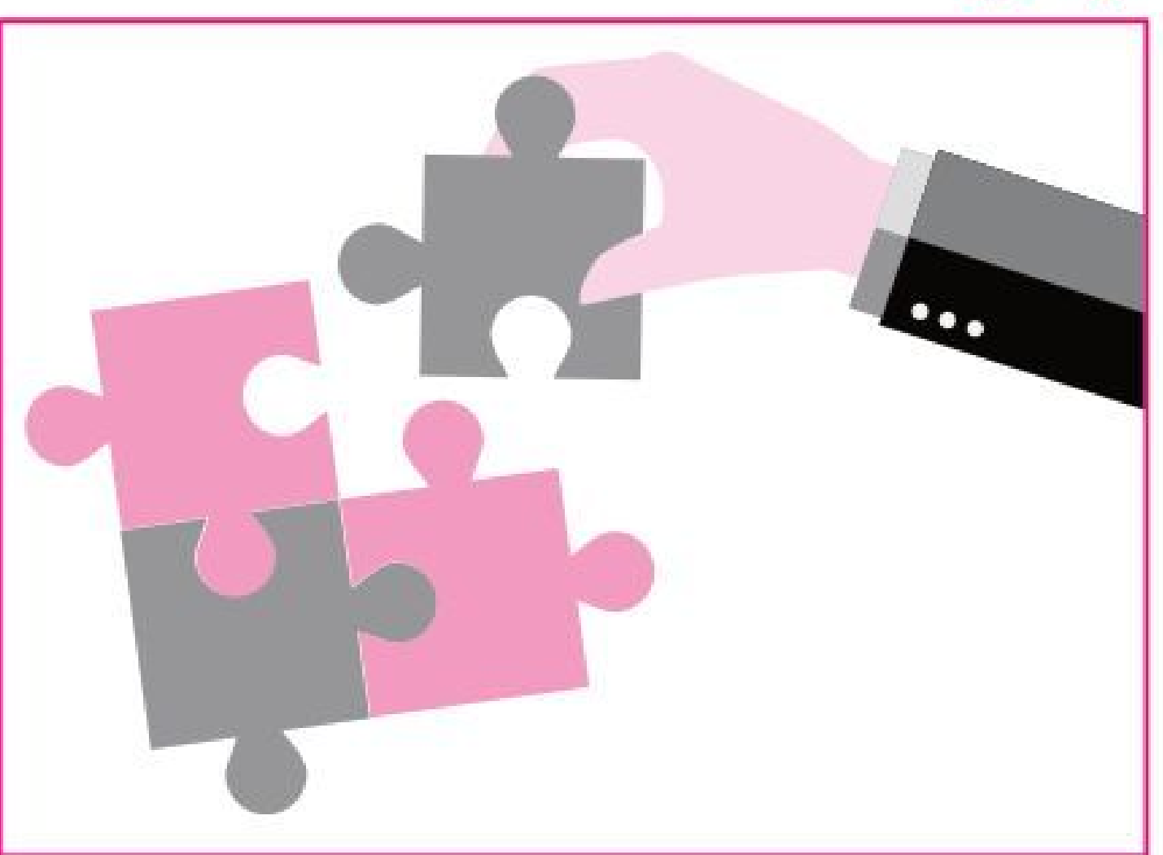
$\therefore$ $f(x) = x^3 - 2x^2 + x + 5$. $f'(x) = 0 \Rightarrow x = \dfrac{1}{3}, 1$

Global max. of $f(x) = 7$

◆◆

PUZZLE CORNER

ANSWER - APRIL 2022



3– 2	5	5× 1	216× 3	6	4
72× 1	13+ 6	5	2– 4	2	3
6	3	4	2÷ 2	1	1– 5
3	4	2÷ 2	1	60× 5	6
8× 4	2	3– 6	11+ 5	3	2÷ 1
5÷ 5	1	3	6	4	2

JEE WORK OUTS

SECTION-I

SINGLE CORRECT OPTION

This section contains 7 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which ONLY ONE is correct. [3 marks for correct answer and –1 for wrong answer]

1. If $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{n}$ and

$$H'_n = \frac{n+1}{2} - \left[\frac{1}{n(n-1)} + \frac{2}{(n-1)(n-2)} + \frac{3}{(n-2)(n-3)} + ... + \frac{n-2}{3\cdot 2}\right],$$

then

(a) $H_n = H'_n$ (b) $2H_n = H'_n$

(c) $H_n = 2H'_n$ (d) $H_n = 3H'_n$

2. If $f(x) = \sin x - x$, then $\int_{-2\pi}^{2\pi} \left|f^{-1}(x)\right| dx$ equal to

(a) π^2 (b) $2\pi^2$ (c) $3\pi^2$ (d) $4\pi^2$

3. Given two circles $x^2 + y^2 + 3\sqrt{2}(x+y) = 0$ and $x^2 + y^2 + 5\sqrt{2}(x+y) = 0$. Let the radius of the third circle, which is tangent to the two given circles and to their common diameter, be $\frac{2p-1}{p}$. The value of p is

(a) 7 (b) 5 (c) 8 (d) 10

4. The equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$, is

(a) $(x-1)^2 + (y-0)^2 = 11/3$

(b) $(x-1)^2 + (y-0)^2 = 22/3$

(c) $(x-1)^2 + (y-0)^2 = 11$

(d) $(x+1)^2 + (y+0)^2 = 11/13$

5. If $z = x + 3i$, then the value of $\int_2^4 \left[\arg\left|\frac{z-i}{z+i}\right|\right] dx$, (where [.] denotes the greatest integer function and $i = \sqrt{-1}$), is

(a) $3\sqrt{2}$ (b) $6\sqrt{3}$

(c) $\sqrt{6}$ (d) None of these

6. Let $\overrightarrow{AB} = 3\hat{i} + \hat{j} - \hat{k}$ and $\overrightarrow{AC} = \hat{i} - \hat{j} + 3\hat{k}$. If the point P on the line segment BC is equidistant from AB and AC then $\overrightarrow{AP}$ is

(a) $2\hat{i} - \hat{k}$ (b) $\hat{i} - 2\hat{k}$

(c) $2\hat{i} + \hat{k}$ (d) None of these

7. If $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$ $= k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then k is equal to

(a) 1 (b) –1 (c) $\alpha\beta$ (d) $\frac{1}{\alpha\beta}$

SECTION-II

MULTIPLE CORRECT OPTION

This section contains 4 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d) out of which ONE OR MORE is/are correct. [4 marks for correct answer and 0 for wrong answer]

8. If $f(x)$ is a cubic polynomial such that $f(0) + 2 = 0$ and $f'''(0) = 6$ and $f(x) = 0$ possess three positive integral roots then

(a) $f'(1) = 0$ (b) $2f'(0) + f''(0) = 0$

(c) $2f'(0) + f''(0) = 2$ (d) $f'(2) = 0$

9. On the interval $A = [-2, 2]$, the function

$$f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

(a) is continuous for all $x \in A - \{0\}$

(b) is continuous for all $x \in A$

(c) assumes all intermediate values from $f(-2)$ to $f(2)$

(d) has maximum value equal to $3/e$

10. The number of real solutions of the equation $3\sqrt{[32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1]^2} = \sin x$ is

(a) $6,\ x \in \left[0, \frac{\pi}{2}\right]$ (b) $12,\ x \in [0, \pi]$

(c) $6,\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $12,\ x \in [-\pi, \pi]$

11. Let $f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$. Suppose $|f(x)| \le 1\ \forall\ x \in [0, 1]$, then

(a) $|a| \le 8$ (b) $|b| \le 8$

(c) $|c| \le 1$ (d) $|a| + |b| + |c| \le 17$

SECTION-III

LINKED COMPREHENSION TYPE

This section contains 2 paragraphs. Based upon one of the paragraphs 2 multiple choice questions and based on the other paragraph 3 multiple choice questions have to be answered. Each of these questions has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct. [3 marks for correct answer and –1 for wrong answer]

Paragraph for Question Nos. 12 and 13

Consider $a_1 + \frac{1}{a_2} + \frac{1}{2a_3} + ... + \frac{1}{(n-1)a_n} = \frac{1}{2a_{n+1}}$; $n \ge 2$, $n \in N$ where $a_1 = 1$, $a_2 = 1/2$.

12. $\sum_{i=1}^{\infty} a_i$ is

(a) 1/2 (b) 1 (c) 2 (d) 3/2

13. $\frac{1}{a_2} + \frac{1}{2a_3} + \frac{1}{3a_4} + ... + \frac{1}{(n-1)a_n} - \frac{1}{3a_3} - \frac{1}{4a_4} - ... - \frac{1}{na_n}$ is equal to

(a) 0 (b) n (c) $n - 1$ (d) $n + 1$

Paragraph for Question Nos. 14 to 16

Two persons A and B are playing with 5 dice. Player A throws 3 dice and Player B throws 2 dice. The trials go on simultaneously and successively until 6 shows on at least one of the dice.

14. $P(A)$: The probability that player A and not B first obtains 6 is

(a) $\frac{(6^2 - 5^2)}{6^5 - 5^5}$ (b) $\frac{(6^3 - 5^3)6^2}{6^5 - 5^5}$

(c) $\frac{(6^3 - 5^3)(6^2 - 5^2)}{6^5 - 5^5}$ (d) None of these

15. $P(B)$: The probability that player B and not A first obtain 6 is

(a) $\frac{(6^2 - 5^2)5^3}{6^5 - 5^5}$ (b) $\frac{(6^3 - 5^3)5^2}{6^5 - 5^5}$

(c) $\frac{(6^3 - 5^3)(6^2 - 5^2)}{6^5 - 5^5}$ (d) None of these

16. $P(C)$: The probability that players A and B obtain a 6 simultaneously is

(a) $\frac{(6^3 - 5^3)5^3}{6^5 - 5^5}$ (b) $\frac{(6^2 - 5^2)5^3}{6^5 - 5^5}$

(c) $\frac{(6^3 - 5^3)(6^2 - 5^2)}{6^5 - 5^5}$ (d) None of these

SECTION-IV

INTEGER ANSWER TYPE

This section contains 7 questions. The answer to each of the questions is a single–digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. [4 marks for correct answer and 0 for wrong answer]

17. If the sum of squares of all integral values of a is S. For which the quadratic expression $(x - a)(x - 10) + 1$ can be factored as a product of two factors $(x + \alpha)(x + \beta)$ and $\alpha, \beta, \in I$, then $1/26S$ is equal to

18. $f(x)$ is a polynomial of 6th degree and $f(x) = f(2 - x)$ $\forall\ x \in \mathbb{R}$. If $f(x) = 0$ has 4 distinct real roots and two real and equal roots then sum of roots of $f(x) = 0$ is

19. If in a rectangular hyperbola $xy = c^2$, the locus of the middle points of chords of constant length $2d$ is $(x^2 + y^2)(xy - c^2) = d^{\lambda}xy$, then the value of λ must be

20. Let the curves $a_1x^2 + 2h_1xy + b_1y^2 - 2g_1x - 2f_1y + c = 0$ and $a_2x^2 - 2h_2xy + (a_2 + a_1 - b_1)y^2 - 2g_2x - 2f_2y + c = 0$ $(a_1 \ne b_1)$ intersect at four concylic points A, B, C and D. If H be the point $\left(\frac{g_1 + g_2}{a_1 + a_2}, \frac{f_1 + f_2}{a_1 + a_2}\right)$, then $\frac{5HA^2 + 7HB^2 - 4HC^2}{HD^2} =$

21. If $\phi^5 = 1, (\phi \ne 1)$, then $\frac{\phi}{1+\phi^2} + \frac{\phi^2}{1+\phi^4} + \frac{\phi^3}{1+\phi} + \frac{\phi^4}{1+\phi^3}$ equals

22. If number of ordered triplets (a, b, c) such that LCM$(a, b) = 1000$, LCM$(b, c) = 2000$ and LCM$(c, a) = 2000$ is α, then sum of the digits in α equals

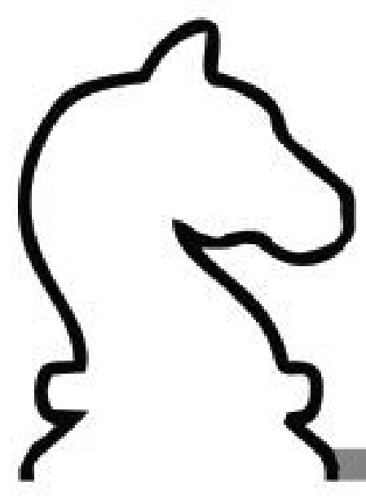
23. Find the number of polynomials $f(x)$ with integral coefficient with $f(a) = b, f(b) = c, f(c) = a$ for distinct integers a, b, c.

ANSWERS

1. (a) **2.** (d) **3.** (c) **4.** (a) **5.** (d)
6. (c) **7.** (a) **8.** (a, c) **9.** (a, c, d)
10. (a, b, c, d) **11.** (a, b, c, d) **12.** (c)
13. (b) **14.** (b) **15.** (a) **16.** (c) **17.** (8)
18. (6) **19.** (2) **20.** (8) **21.** (2) **22.** (7)
23. (0)

◆

Challenging PROBLEMS ON JEE Main



1. Let $f(x)=$ $\int (x-{}^{10}C_1x^2+{}^{10}C_2x^3-{}^{10}C_3x^4+\ldots+{}^{10}C_{10}x^{11})dx$ and $f(0)=\frac{263}{132}$ then $|f(1)|=$

(a) 1 (b) 2 (c) 3 (d) 4

2. If $f_1(x)$ and $f_2(x)$ are two antiderivatives of $f(x)=ax+b+\sin x$ such that $\int_2^4 (f_1(x)-f_2(x))dx=8$ then, the area of triangle formed by the points (0, 0), $(3, f_1(3))$ and $(3, f_2(3))$ is

(a) 2 (b) 4 (c) 6 (d) 8

3. Let $p(x)$ be a continuous function, which is positive for all $x\in R$ and $\int_2^3 p(x)dx=c\cdot\int_0^2 p\left(\frac{x+4}{2}\right)dx$ then $2c+1=$

(a) 0 (b) 1 (c) 2 (d) 3

4. $$f(x)=\begin{vmatrix} x^3-4x & \sin x-\tan x & \sec^2 x \\ 3a^2+1 & 2a & a^6+6 \\ a+\lambda & 1 & 2 \end{vmatrix}$$

If $\int_{-\pi/4}^{\pi/4} f(x)dx\ge 0\ \forall\ a\in R$, then number of integral values of λ is/are

(a) 0 (b) 1

(c) 2 (d) 3

5. If $f(x)=|||3\cos x-\sin x|+2\cos x|+\cos x|,\ x\in\left(\frac{\pi}{2},\pi\right)$ then $\int_{\pi/2}^{\pi} f(x)dx=$

(a) 0 (b) 1

(c) −1 (d) π

6. $\int_0^{\infty}\frac{x^2+1}{x^4-x^2+1}dx=$

(a) 0 (b) 1 (c) π (d) π/2

7. $f(x)=\cos 2x\cos 4x\cos 8x\cos 16x\cos 32x$, $\int_{-\pi}^{\pi} f(x)dx=$

(a) π (b) 2π (c) π/2 (d) 0

8. Consider the function, $f(\theta)=\int_0^1\frac{|\sqrt{1-x^2}-\sin\theta|}{\sqrt{1-x^2}}dx$, $\theta\in\left[0,\frac{\pi}{2}\right]$ then max. $f(\theta)$ + min. $f(\theta)$ =

(a) 1 (b) $\sqrt{2}$ (c) $\frac{\pi}{2}-1$ (d) $2-\frac{\pi}{2}$

9. Consider, $f(x)=\int_0^x\left(\frac{f(t)}{1+e^{t-x}}+\frac{f(x-t)}{1+e^t}+e^t\right)dt$. The number of solutions of the equation $f(x)+x^2=0$ is/are

(a) 0 (b) 2 (c) 4 (d) 6

10. If $I=\int_0^{102}[\cot^{-1}x]dx$, then $[I]=$

(where [·] denotes greatest integer function)

(a) 0 (b) 1 (c) −1 (d) −2

11. If $f(x)=x^5+5x-1$ then $\int_5^{41}\frac{dx}{5f^{-1}(x)+(f^{-1}(x))^5}=$

(a) ln1 (b) ln3 (c) ln5 (d) ln7

12. $f(x)=\int\frac{(1+x)((1-x+x^2)(1+x+x^2)+x^2)}{(1+2x+3x^2+4x^3+3x^4+2x^5+x^6)}dx$, $[f(99)-f(3)]=$

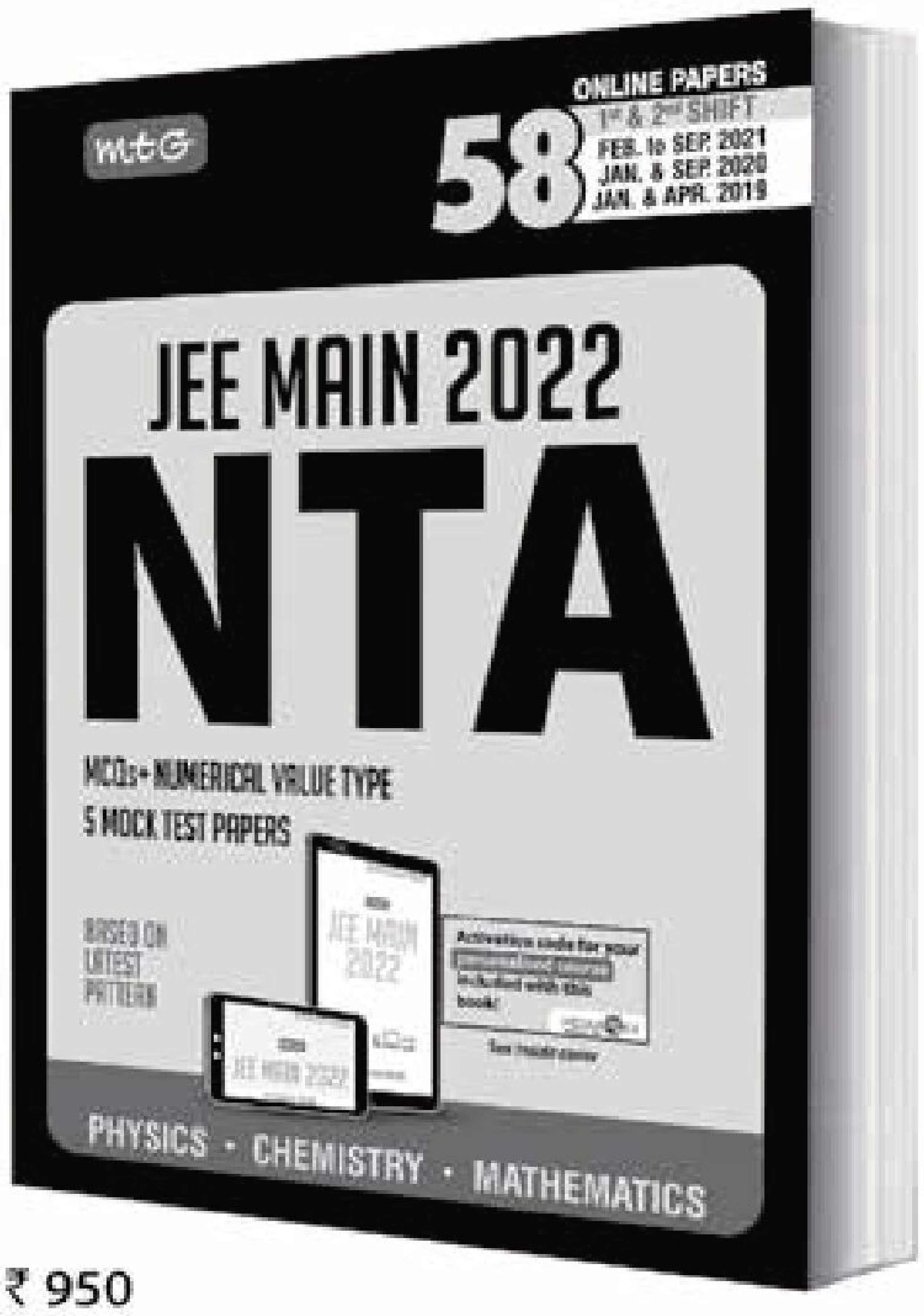
(where [·] denotes greatest integer function)

(a) 0 (b) 1 (c) 2 (d) 3

By : Tapas Kr. Yogi, Visakhapatnam Mob : 09533632105

mtG

Reach the peak of readiness for JEE Main 2022


mtG
58
ONLINE PAPERS
1st & 2nd SHIFT
FEB. to SEP. 2021
JAN. & SEP. 2020
JAN. & APR. 2019
JEE MAIN 2022
NTA
MCQs+ NUMERICAL VALUE TYPE
5 MOCK TEST PAPERS
BASED ON LATEST PATTERN
PHYSICS · CHEMISTRY · MATHEMATICS

₹ 950

The Wait is Over! MTG Presents you all the 26 Fully Solved Question Papers of JEE Main 2021 Session **(I)**, **(II)**, **(III)** **and** **(IV)**; 16 Fully Solved Question Papers of **JEE Main (I) January and JEE Main (II) September 2020** & 16 Fully Solved Question Papers of **JEE Main (I) January** and **JEE Main (II) April, 2019** conducted by NTA with Chapterwise graphical analysis showing the weightage of chapters and Mock Test Papers as per latest pattern of JEE Main. Get the maximum benefit from the book through personalised course on **Pedagogy app**. The best preparation for tomorrow is doing your best today so, go and get your copy.

Highlights

- Fully Solved Authentic Papers
- **26** (Feb, March, July & Sept. 2021) + **16** (Jan. & Sept., 2020) + **16** (Jan. & Apr., 2019) Online Papers
- 4915 MCQs for Practice
- Chapterwise Tabular and Graphical Analysis Showing the Weightage of Chapters
- 5 MTPs as per the latest JEE Main pattern

Practice Online Tests to Predict Your Rank

mtG

Make It Happen, CRACK JEE Main

Get the complete feel of Online exam by attempting tests on computers

Take the Final Leap Towards ***JEE Main*** Success!

Available at all leading book shops throughout the country. To buy online visit www.mtg.in.

For more information or for help in placing your order, call 0124-6601200 or email: *info@mtg.in*

13. Let $\int e^x(x^2-1)(x^2+4x+1)dx = e^x f(x)+c$. If α is one irrational root of $f(x) + f'(x) = 0$ then value of $\alpha^4 + 4\alpha^3 + 2\alpha^2 + 4\alpha =$

(a) 0 (b) −1 (c) 1 (d) 2

14. $\int_{-1}^{1} e^{-x^4}(1+5x^3-4x^4+\ln(x+\sqrt{x^2+1}))dx =$

(a) 0 (b) 1 (c) $\frac{1}{e}$ (d) $\frac{2}{e}$

15. $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)^{3/2}} =$

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

16. $\int_{-1}^{1} \frac{dx}{x^2+x+1+\sqrt{x^4+3x^2+1}} =$

(a) 0 (b) $\pi/2$ (c) $\pi/4$ (d) π

17. Let, $I = \int_0^1 \frac{dx}{1+x^{\pi/2}}$, then

(a) $I \in (\ln 2, \pi/4)$ (b) $\ln 2 > I$
(c) $I = \pi/4$ (d) $I = \ln(2)$

18. If $\int_0^{\infty} \frac{\ln(bx)}{x^2+a^2}dx = \frac{\pi}{2a}$ $(a, b > 0)$ then $ab =$

(a) 1 (b) e (c) $\frac{1}{e}$ (d) $\frac{2}{e}$

19. Let $I_n = \int_0^{\pi/2} (1-\sin x)^n \sin 2x dx$, $\sum_{n=0}^{\infty} I_n =$

(a) 1 (b) 2 (c) 4 (d) 8

20. If $\int_{-m}^{m} \frac{3\{x\}+1}{\{3x\}+1}dx = 6\ln(4e^2)$ then $m =$

(a) 1 (b) 2 (c) 3 (d) 6

SOLUTIONS

1. (b) : $f(x) = \int x(1-x)^{10}dx$

Putting $1 - x = t$, we get

$f(1 - t) = -\int(1 - t)t^{10}dt \Rightarrow f(1 - t) = \int t^{11}dt - \int t^{10}dt$

$$\Rightarrow f(1-t) = \frac{t^{12}}{12} - \frac{t^{11}}{11} + c$$

Now, put $t = 1$, we get

$$f(0) = \frac{1}{12} - \frac{1}{11} + c \Rightarrow c = 2 \quad \therefore \; f(1-t) = \frac{t^{12}}{12} - \frac{t^{11}}{11} + 2$$

Put $t = 0$, we get

$$|f(1)| = |0-0+2| = |2| = 2$$

2. (c) : Notice that $f_1(x) - f_2(x) = c_1 - c_2$

So, $\int_2^4 (c_1-c_2)dx = 8 \Rightarrow c_1 - c_2 = 4$

and required area of $\Delta = \frac{1}{2} \times 3 \times |f_1(3) - f_2(3)|$

$$= \frac{1}{2} \times 3 \times 4 = 6$$

3. (c) : Put, $\frac{x+4}{2} = y$, $\int_2^3 p(x)(1-2c)dx = 0$

As $p(x) > 0 \Rightarrow 1 - 2c = 0 \Rightarrow 2c = 1 \Rightarrow 2c + 1 = 2$

4. (d) : Expanding $f(x)$ by 1st row, we have

$$f(x) = \underbrace{(x^3-4x)()}_{\text{odd}} - \underbrace{(\sin x - \tan x)()}_{\text{odd}}$$
$$+ \underbrace{\sec^2 x(3a^2+1-2a(a+\lambda))}_{\text{even}}$$

$\therefore \int_{-\pi/4}^{\pi/4} f(x)dx \geq 0$ becomes,

$$2\int_0^{\pi/4} \sec^2 x(a^2-2a\lambda+1)dx \geq 0$$

i.e., $a^2 - 2a\lambda + 1 \geq 0 \; \forall \; a \in R$

$\Rightarrow \lambda \in [-1, 1]$ for $D \leq 0$

5. (b) : $f(x) = ||\sin x - 3\cos x + 2\cos x| + \cos x| = \sin x$

$\therefore \int_{\pi/2}^{\pi} \sin x \, dx = (-\cos x)_{\pi/2}^{\pi}$

$$= -\cos\pi + \cos\frac{\pi}{2} = -(-1) + 0 = 1$$

6. (c) : $I = \int_0^{\infty} \frac{x^2+1}{x^4-x^2+1}dx = \int_0^{\infty} \frac{1+\frac{1}{x^2}}{x^2-1+\frac{1}{x^2}}dx$

Putting $\left(x - \frac{1}{x}\right) = t$, we get

$$I = \int_{-\infty}^{\infty} \frac{dt}{t^2+1} = 2\int_0^{\infty} \frac{dt}{t^2+1} = 2\tan^{-1}t\,|_0^{\infty} = \pi$$

7. (d) : $f(x)$ is even. So, $I = 2 \cdot \int_0^{\pi} f(x)dx$

$I = 2 \cdot 2\int_0^{\pi/2} f(x)dx$ as $f(x)$ is symmetric about $x = \pi/2$

So, $I = 4\cdot\int_0^{\pi/2} f(x)dx = 4I'(\text{say})$

Now, $I' = \int_0^{\pi/2} f(x)dx = -\int_0^{\pi/2} f(x)dx,$

(Using $f(a-x)$ property)

$\Rightarrow$ $I' = -I' \Rightarrow I' = 0$. Hence, $I = 0$.

8. (b) : Put, $x = \cos\alpha$,

$$f(\theta) = \int_0^{\theta}(\sin\theta - \sin\alpha)d\alpha + \int_{\theta}^{\pi/2}(\sin\alpha - \sin\theta)d\alpha$$

$$= 2(\theta\sin\theta + \cos\theta) - \frac{\pi}{2}\sin\theta - 1$$

So, $f'(\theta) = 2\left(\theta - \frac{\pi}{4}\right)\cos\theta$

$\Rightarrow$ f is decreasing in $\left(0, \frac{\pi}{4}\right)$ and increasing in $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

So, max $f(\theta) = 1$, min $f(\theta) = \sqrt{2} - 1$

9. (b) : Using $f(a-x)$ property, we have

$$f(x) = \int_0^x\left(\frac{f(t)}{1+e^{t-x}} + \frac{f(t)}{1+e^{x-t}} + e^t\right)dt$$

i.e., $f(x) = \int_0^x (f(t) + e^t)dt$

$\Rightarrow$ $f'(x) = f(x) + e^x$

and $f(0) = 0$ so, $f(x) = xe^x$.

So, solutions to equation $xe^x + x^2 = 0$ is $x = 0$ and one more from $e^x = -x$. Hence it has total 2 solutions.

10. (a) : $I = \int_0^{\cot 1} 1dx + \int_{\cot 1}^{102} 0dx = \cot 1 < 1 \Rightarrow [I] = 0$

11. (d) : $f(x) = x^5 + 5x - 1$ (in c.)

Let $f^{-1}(x) = t$. So, $f^{-1}(5) = 1$, $f^{-1}(41) = 2$

$\therefore \int_1^2 \frac{5t^4+5}{t^5+5t}dt = \ln 42 - \ln 6 = \ln 7 + \ln 6 - \ln 6 = \ln 7$

12. (d) : $f(x) = \int\frac{(1+x)(1+x^2)^2}{(1+x)^2(1+x^2)^2}dx = \ln(1+x) + c$

$[f(99) - f(3)] = [\ln 100 - \ln 4] = [\ln 25] = [2 \times \ln 5]$
$= [3.218] = 3$

13. (b) : Differentiating the given integral, we have
$(x^2-1)(x^2+4x+1) = f(x) + f'(x)$

$\Rightarrow$ α is a root of $x^2 + 4x + 1 = 0$

i.e., $\alpha^2 + 4\alpha + 1 = 0 \Rightarrow \alpha^2 + 4\alpha = -1$

Now, $\alpha^4 + 4\alpha^3 + 2\alpha^2 + 4\alpha$

$= \alpha^2(\alpha^2 + 4\alpha) + \alpha^2 + (\alpha^2 + 4\alpha) = -\alpha^2 + \alpha^2 + (-1) = -1$

14. (d) : Notice that $5x^3$ and $\ln(x + \sqrt{x^2+1})$ are odd functions

So, $I = 2\cdot\int_0^1 (1-4x^4)\cdot e^{-x^4}dx = 2\cdot\int_0^1 \frac{d}{dx}(xe^{-x^4})dx = \frac{2}{e}$

15. (c) : $I = 2\int_0^{\infty}\frac{dx}{(x^2+4)^{3/2}}$

Putting $x = 2\tan\theta$, we get $I = \frac{1}{2}\int_0^{\pi/2}\cos\theta d\theta = \frac{1}{2}$

16. (c) : Rationalizing, $I = 2\int_0^1\frac{dx}{2(1+x^2)} = \tan^{-1}x\Big|_0^1 = \frac{\pi}{4}$

17. (a) : For $x \in (0, 1)$, $x^2 < x^{\pi/2} < x$

$$\Rightarrow \frac{1}{1+x} < \frac{1}{1+x^{\pi/2}} < \frac{1}{1+x^2}.$$

$$\Rightarrow \int_0^1\frac{1}{1+x}dx < \int_0^1\frac{1}{1+x^{\pi/2}}dx < \int_0^1\frac{1}{1+x^2}dx$$

$\Rightarrow \ln 2 < I < \frac{\pi}{4}$

18. (b) : Putting : $x = a\tan\theta$, we get

$$I = \frac{1}{a}\int_0^{\pi/2}(\ln(ab) + \ln(\tan\theta))d\theta$$

$$= \frac{\pi}{2a}\ln(ab) + \frac{1}{a}\cdot(0)$$

$\Rightarrow$ $\ln(ab) = 1 \Rightarrow ab = e$

19. (b) : Putting $\sin x = t$, we get

$$I_n = \int_0^1 2(1-t)^n\cdot t dt = 2\left(\int_0^1 t^n dt - \int_0^1 t^{n+1}dt\right)$$

$\Rightarrow I_n = 2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$. So, $\sum_{n=0}^{\infty} I_n = 2(1) = 2$

20. (d) : $I_m = 2m\int_0^1\frac{3\{x\}+1}{\{3x\}+1}dx$

$$= 2m\cdot\left[\int_0^{1/3}\frac{3x+1}{3x+1}dx + \int_{1/3}^{2/3}\frac{3x+1}{3x-1+1}dx + \int_{2/3}^{1}\frac{3x+1}{3x-2+1}dx\right]$$

$= m\ln(4e^2)$. $\quad\therefore m = 6$

◈◈

CUET (UG)

PRACTICE PAPER 2022

Section II of CUET (UG) is Domain specific. In this section of Mathematics 40 questions to be attempted out of 50.

Time : 45 minutes

1. If $y = (\cos x^2)^2$, then $\dfrac{dy}{dx}$ is equal to
(a) $-4x \sin 2x^2$ (b) $-x \sin x^2$
(c) $-2x \sin 2x^2$ (d) $-x \cos 2x^2$

2. The differential equation of all parabolas whose axis is y-axis is
(a) $x\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = 0$ (b) $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = 0$
(c) $\dfrac{d^2y}{dx^2} - y = 0$ (d) $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} = 0$

3. The rate of change of area of a circle with respect to its radius at $r = 2$ cm is
(a) 4 (b) 2π (c) 2 (d) 4π

4. $\int e^{-x} \operatorname{cosec} x (1 + \cot x)\, dx =$
(a) $e^{-x} \operatorname{cosec} x + C$
(b) $-e^{-x} \operatorname{cosec} x + C$
(c) $-e^{-x}(\operatorname{cosec} x + \cot x) + C$
(d) $-e^{-x}(\operatorname{cosec} x - \tan x) + C$

5. If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then the value of $|\vec{b}|$ is
(a) 1 (b) 2 (c) 3 (d) 4

6. If $A = \{a, b, c\}$, then the number of binary operations on A is
(a) 3^6 (b) 3^3 (c) 3^9 (d) 3

7. If $\cos \alpha$, $\cos \beta$, $\cos \gamma$ are the direction cosines of a vector $\vec{a}$, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to,
(a) 2 (b) 3 (c) -1 (d) 0

8. If $\begin{bmatrix} \sin^2 \alpha & \cos^2 \alpha \\ \cos^2 \alpha & \sin^2 \alpha \end{bmatrix} = 0, \alpha \in (0, \pi)$, then the values of α are
(a) $\dfrac{\pi}{2}$ and $\dfrac{\pi}{12}$ (b) $\dfrac{\pi}{2}$ and $\dfrac{\pi}{6}$
(c) $\dfrac{\pi}{4}$ and $\dfrac{3\pi}{4}$ (d) $\dfrac{\pi}{6}$ and $\dfrac{\pi}{3}$

9. For the following distribution function $F(X)$ of a random variable X

X	1	2	3	4	5	6
$F(X)$	0.2	0.37	0.48	0.62	0.85	1

$P(3 < X \le 5) =$
(a) 0.48 (b) 0.37 (c) 0.27 (d) 1.47

10. Area bounded by $y = x^3$, $y = 8$ and $x = 0$ is ______.
(a) 14 sq. units (b) 6 sq. units
(c) 2 sq. units (d) 12 sq. units

11. If $f(x) = \begin{cases} \log(\sec^2 x)^{\cot^2 x} & , \text{ for } x \ne 0 \\ K & , \text{ for } x = 0 \end{cases}$
is continuous at $x = 0$, then K is
(a) e^{-1} (b) 1 (c) e (d) 0

12. If $A^{-1} = \dfrac{1}{11}\begin{bmatrix} -3 & 4 \\ 5 & -3 \end{bmatrix}$, then $A =$
(a) $\dfrac{-1}{11}\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$ (b) $\dfrac{1}{11}\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$
(c) $\begin{bmatrix} 3 & -4 \\ -5 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}$

13. The domain of the function
$f(x) = \sin^{-1}\left(\dfrac{x+5}{2}\right)$ is
(a) $[-1, 1]$ (b) $[2, 3]$
(c) $[3, 7]$ (d) $[-7, -3]$

14. If $\begin{bmatrix} x+y & x-y \\ 2x+z & x+z \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, then the values of x, y and z are respectively
(a) 0, 0, 1 (b) 1, 1, 0 (c) -1, 0, 0 (d) 0, 0, 0

15. If $n \in N$ and $I_n = \int (\log x)^n dx$, then $I_n + nI_{n-1} =$
(a) $\dfrac{(\log x)^n}{n}$ (b) $\dfrac{(\log x)^{n+1}}{n+1}$
(c) $x(\log x)^n + C$ (d) $(\log x)^{n-1}$

16. The value of $\tan\left[\sin^{-1}\frac{5}{13}+\cot^{-1}\frac{4}{3}\right]$ is

(a) $\frac{26}{11}$ (b) $\frac{56}{33}$ (c) $\frac{63}{41}$ (d) $\frac{65}{43}$

17. If the function $f(x)$ defined by

$$f(x)=\begin{cases} x\sin\frac{1}{x}, & \text{for } x\neq 0 \\ k, & \text{for } x=0 \end{cases}$$

is continuous at $x = 0$, then $k =$ ________

(a) 0 (b) 1 (c) – 1 (d) 1/2

18. The perimeter of a sector is a constant. If its area is to be maximum, then the sectorial angle is

(a) 2° (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) 4°

19. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+3\hat{j}+5\hat{k}$ and $\vec{c}=7\hat{i}+9\hat{j}+11\hat{k}$. Then the area (in sq. units) of the parallelogram with diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is

(a) $4\sqrt{6}$ (b) $\frac{1}{2}\sqrt{21}$ (c) $\frac{\sqrt{6}}{2}$ (d) $\sqrt{6}$

20. The interval in which the function $f(x) = x^3 - 6x^2 + 9x + 10$ is increasing in

(a) $(-\infty, 1) \cup (3, \infty)$ (b) $[1, 3]$

(c) $(-\infty, 1] \cup [3, \infty)$ (d) $(-\infty, -1] \cup [3, \infty)$

21. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1) = 1$, then $y(1/2)$ is equal to

(a) 7/64 (b) 49/16

(c) 1/4 (d) 13/16

22. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC is at the point (1, 2, 3) then the equation of the plane is

(a) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ (b) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

(c) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=\frac{1}{3}$ (d) $\frac{x}{1}-\frac{y}{2}+\frac{z}{3}=-1$

23. If $A=\begin{bmatrix}2 & 2\\ -3 & 2\end{bmatrix}, B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$ then $(B^{-1}A^{-1})^{-1}$ = _____

(a) $\begin{bmatrix}2 & -2\\ 2 & 3\end{bmatrix}$ (b) $\begin{bmatrix}2 & 2\\ -2 & 3\end{bmatrix}$

(c) $\begin{bmatrix}2 & -3\\ 2 & 2\end{bmatrix}$ (d) $\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$

PUBLIC NOTICE CUET (UG) 2022

Sub : Conduct of the Common University Entrance Test CUET (UG)-2022 for Under Graduate Programmes in Central Universities for the Academic Session 2022-23

National Testing Agency has been entrusted with the responsibility of conducting the Undergraduate entrance tests for all the Central Universities (CUs) for the academic session 2022-2023. CUET will provide a single window opportunity to students to seek admission in any of the Central Universities (CUs) across the country.

The CUET (UG) - 2022 will be conducted in Computer Based Test (CBT) Mode. The aspirants who desire to appear for the test, may refer to the Information Bulletin for admission to Undergraduate programmes of the desired Central Universities (CUs) for admission into the Undergraduate Programmes.

The details of the programmes offered by Central Universities (CUs) are being made available on their respective websites.

Examination Structure for CUET (UG) - 2022

CUET (UG) – 2022 will consist of the following 4 Sections :

- Section IA – 13 Languages
- Section IB – 20 Languages
- Section II – 27 Domain Specific Subjects
- Section III – General Test

Choosing options from each Section is not mandatory. Choices should match the requirements of the desired University.

- A Candidate can choose a maximum of **any 3 languages** from Section IA and Section IB taken together. However, the third language chosen needs to be in lieu of 6th domain specific subject chosen by the candidate - as applicable (**so the maximum number of tests to be taken remains 9 only *i.e.* 2 Languages + 6 Domain Specific Subjects + 1 General Test OR 3 Languages + 5 Domain Specific Subjects + 1 General Test:** flexibility being provided to help a candidate apply for many Universities depending on their eligibility conditions).
- Section II offers 27 Subjects, out of which a candidate may choose a maximum of 6 Subjects.
- Section III comprises **General Test**.
- For choosing Languages from Section IA and IB and domain specific Subjects from Section II and General Test under Section III, the Candidate must refer to the requirements of his/her intended University.

Mode of the Test	Computer Based Test-CBT
Test Pattern	Objective type with Multiple Choice Questions
Medium	13 languages (*Tamil, Telugu, Kannada, Malayalam, Marathi, Gujarati, Odiya, Bengali, Assamese, Punjabi, English, Hindi and Urdu*)
Syllabus	**Section IA & IB:** Language to be tested through Reading Comprehension (based on different types of passages– Factual, Literary and Narrative [Literary Aptitude & Vocabulary]
	Section II : As per NCERT model syllabus as applicable to Class XII only
	Section III : General Knowledge, Current Affairs, General Mental Ability, Numerical Ability, Quantitative Reasoning (Simple application of basic mathematical concepts arithmetic/algebra geometry/mensuration/stat taught till Grade 8), Logical and Analytical Reasoning
Registration	Registration will be online at **https://cuet.samarth.ac.in/.**

Schedule of Examination

Examination will be conducted on multiple days in two shifts, depending on the number of Candidates and Languages/Tests taken by them.

24. If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4}(3-f(x))dx = 7,$ then $\int_{-1}^{2} f(x)dx$ is

(a) 5 (b) 2 (c) 3 (d) 4

25. If random varaible $X \sim B\left(n=5, p=\frac{1}{3}\right),$ then $P(2 < X < 4) =$

(a) $\frac{80}{243}$ (b) $\frac{40}{243}$ (c) $\frac{40}{343}$ (d) $\frac{80}{343}$

26. The value of $10\tan(\cot^{-1}3 + \cot^{-1}7)$ is equal to

(a) 3 (b) 5 (c) 7 (d) 9

27. The area bounded by $y = x + 2$, $y = 2 - x$ and the x-axis is (in square units)

(a) 1 (b) 2 (c) 4 (d) 6

28. If $\tan^{-1}(x^2 + y^2) = \alpha$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{-x}{y}$ (b) xy (c) $\frac{x}{y}$ (d) $-xy$

29. $f(x) = \begin{cases} 2a - x, & \text{when} - a < x < a \\ 3x - 2a, & \text{when}\ a \le x \end{cases}$

Then which of the following is true?

(a) $f(x)$ is not differentiable at $x = a$.
(b) $f(x)$ is discontinuous at $x = a$.
(c) $f(x)$ is continuous for all $x < a$.
(d) $f(x)$ is differentiable for all $x \ge a$.

30. The value of x satisfying the equation

$$\begin{vmatrix} \cos 2x & \sin 2x & \sin 2x \\ \sin 2x & \cos 2x & \sin 2x \\ \sin 2x & \sin 2x & \cos 2x \end{vmatrix} = 0 \text{ and } x \in \left[0, \frac{\pi}{4}\right] \text{ is}$$

(a) $\pi/8$ (b) $\pi/2$ (c) $\pi/16$ (d) $\pi/3$

31. In the group $G = \{1, 2, 3, 4, 5, 6\}$ under $\otimes_7$, the solution of $4 \otimes_7 x = 5$ is

(a) 5 (b) 3 (c) 2 (d) 4

32. If $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ then the values of x and y respectively are

(a) $-3, -1$ (b) $1, 3$ (c) $3, 1$ (d) $-1, 3$

33. $\int_{0}^{\pi/2} \frac{\tan^7 x}{\cot^7 x + \tan^7 x}dx$ is equal to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

34. If $3A + 4B' = \begin{bmatrix} 7 & -10 & 17 \\ 0 & 6 & 31 \end{bmatrix}$

and $2B - 3A' = \begin{bmatrix} -1 & 18 \\ 4 & 0 \\ -5 & -7 \end{bmatrix}$ then $B =$

(a) $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -18 \\ 4 & -16 \\ -5 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -3 \\ -1 & 1 \\ 2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 3 \\ -1 & 1 \\ 2 & -4 \end{bmatrix}$

35. The two vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 5\hat{k}$ represents the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of a ΔABC. The length of the median through A is

(a) 14 (b) 7 (c) $\sqrt{14}$ (d) $\frac{\sqrt{14}}{2}$

36. If planes $\vec{r}\cdot(p\hat{i} - \hat{j} + 2\hat{k}) + 3 = 0$ and $\vec{r}\cdot(2\hat{i} - p\hat{j} - \hat{k}) - 5 = 0$ include angle $\frac{\pi}{3}$, then the value of p is

(a) $1, -3$ (b) $-1, 3$ (c) -3 (d) 3

37. Two events A and B will be independent if

(a) $P(A' \cap B') = (1 - P(A))\ (1 - P(B))$
(b) A and B are mutually exclusive
(c) $P(A) + P(B) = 1$
(d) $P(A) = P(B)$

38. Inverse of a diagonal non-singular matrix is a

(a) diagonal matrix (b) scalar matrix
(c) skew-symmetric matrix
(d) zero matrix

39. $\cos^{-1}\left(\cos\left(\frac{7\pi}{5}\right)\right) =$

(a) $\frac{3\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{-7\pi}{5}$ (d) $\frac{7\pi}{5}$

40. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are respectively

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(c)	**5.**	(d)
6.	(a)	**7.**	(a,d)	**8.**	(a,d)	**9.**	(b,d)	**10.**	(b,c)
11.	(a,b)	**12.**	(b,d)	**13.**	(d)	**14.**	(b)	**15.**	(c)
16.	(b)	**17.**	(5)	**18.**	(4)	**19.**	(3)	**20.**	(1)

mtG

Success Guaranteed in CUET (UG)

Common University Entrance Test Under-Graduate Programmes with MTG CUET (UG) Practice Papers

From the academic session 2022-23, admissions to Undergraduate Programmes in all 45 central universities in the country will take place through Central University Entrance Test (CUET).

To help students with their preparation for the above test MTG is coming up with the book CUET 2022 which caters the Science Stream for the test. The MTG CUET (UG) – 2022 title covers 15 Practice papers which include following subjects in each practice paper :

Section IA Languages : English and Hindi

Section II Domain : Physics, Chemistry, Mathematics and Biology

Section III General Test : General Knowledge, Current Affairs, General Mental Ability, Numerical Ability, Quantitative Reasoning, Logical and Analytical Reasoning

2022
mtG
CUET (UG)
COMMON UNIVERSITY ENTRANCE TEST
UNDER-GRADUATE PROGRAMMES
15 PRACTICE PAPERS
Includes:
Section-I: Languages
English & Hindi
Section-II: Domain
Physics, Chemistry, Mathematics & Biology
Section-III: General Test
COMPUTER BASED TEST

₹ 800/-

mtG Learning Media

Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Visit **www.mtg.in** to buy online!
Call 0124-6601200 or e-mail:info@mtg.in

Visit
www.mtg.in
for latest offers
and to buy
online!

(a) $1, \frac{2}{3}$ (b) $3, 1$ (c) $1, 2$ (d) $3, 3$

41. Area bounded by the curves $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$ is (in sq. units)

(a) $e+\frac{1}{e}$ (b) $e+\frac{1}{e}+2$

(c) $e+\frac{1}{e}-2$ (d) $e-\frac{1}{e}+2$

42. If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$, where n is positive integer, then $I_{10} + I_8$ is equal to

(a) 9 (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{9}$

43. If $f(x) = 3x + 5$ and $g(x) = x^2 - 1$, then $(fog)(x^2 - 1)$ is equal to

(a) $3x^4 - 3x + 5$ (b) $3x^4 - 6x^2 + 5$

(c) $6x^4 + 3x^2 + 5$ (d) $6x^4 - 6x + 5$

44. A balloon which always remains spherical is being inflated by pumping in 10 cubic centimeters of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm.

(a) $\frac{1}{9\pi}$ cm/sec (b) $\frac{1}{\pi}$ cm/sec

(c) $\frac{1}{90\pi}$ cm/sec (d) $\frac{1}{30\pi}$ cm/sec

45. The equation of the tangent to the curve $y = x^3 - 6x + 5$ at $(2, 1)$ is

(a) $6x - y - 11 = 0$ (b) $6x - y - 13 = 0$

(c) $6x + y + 11 = 0$ (d) $6x - y + 11 = 0$

46. A and B are two events such that $P(A) \neq 0$, $P(B|A)$

(i) A is a subset of B

(ii) $A \cap B = \phi$

are respectively

(a) 1, 1 (b) 0, 1

(c) 0, 0 (d) 1, 0

47. Equation of the plane perpendicular to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and passing through the point $(2, 3, 4)$ is

(a) $2x + 3y + z = 17$ (b) $x + 2y + 3z = 9$

(c) $3x + 2y + z = 16$ (d) $x + 2y + 3z = 20$

48. The solution of the differential equation

$$\frac{x\frac{dy}{dx}-y}{\sqrt{x^2-y^2}}=10x^2 \text{ is}$$

(a) $\sin^{-1}\left(\frac{y}{x}\right)-5x^2=C$

(b) $\sin^{-1}\left(\frac{y}{x}\right)=10x^2+C$

(c) $\frac{y}{x}=5x^2+C$

(d) $\sin^{-1}\left(\frac{y}{x}\right)=10x^2+Cx$

49. The value of $\sin(2\sin^{-1}0.8)$ is equal to

(a) 0.48 (b) sin 1.2°

(c) sin 1.6° (d) 0.96

50. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the inverse of the matrix $\begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix}$, then d equals

(a) $\frac{-1}{38}$ (b) $\frac{-7}{38}$ (c) $\frac{3}{38}$ (d) $\frac{5}{38}$

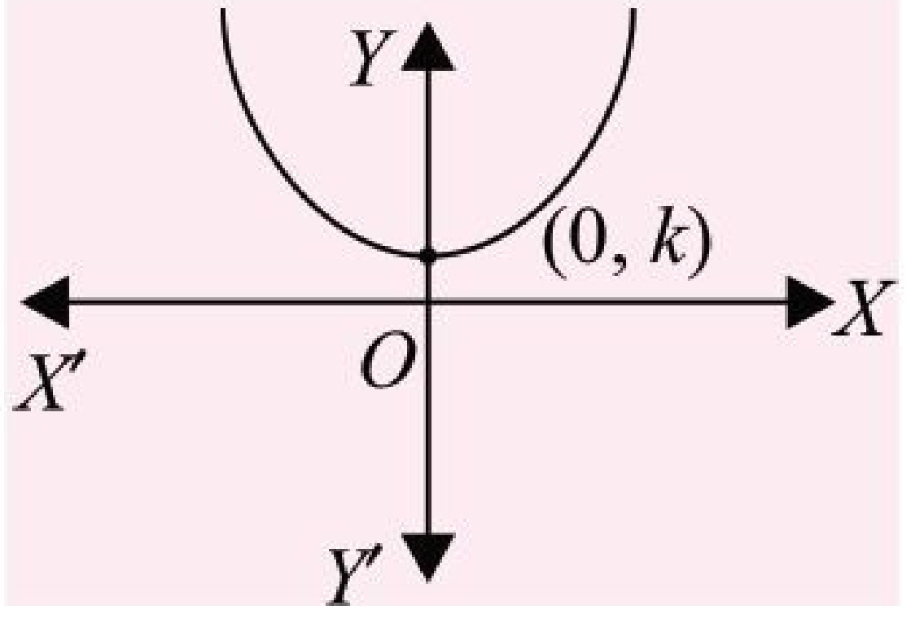
SOLUTIONS

1. (c) : We have, $y = (\cos x^2)^2$

$$\Rightarrow \frac{dy}{dx} = 2\cos x^2 \,.\, (-\sin x^2)2x$$

$$= -2x\sin(2x^2)$$

2. (a) : Consider a parabola having its vertex at (0, k) and axis along y-axis.



Equation of parabola is

$(x - 0)^2 = 4a(y - k)$

$\Rightarrow x^2 = 4ay - 4ak$

Differentiating w.r.t x, we get

$$2x = 4a\frac{dy}{dx} \Rightarrow x = 2a\frac{dy}{dx} \Rightarrow \frac{1}{2a}=\frac{1}{x}\frac{dy}{dx}$$

Again differentiating w.r.t. x, we get

$$\frac{d}{dx}\left(\frac{1}{x}\cdot\frac{dy}{dx}\right)=\frac{d}{dx}\left(\frac{1}{2a}\right)$$

$$\Rightarrow \frac{1}{x}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(-\frac{1}{x^2}\right)=0$$

$$\Rightarrow x\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$$ is the required differential equation.

3. (d) : Area of circle, $A = \pi r^2$ ...(i)

Differentiate (i) w.r.t. r, we get $\frac{dA}{dr}=2\pi r$

$$\therefore \left(\frac{dA}{dr}\right)_{r=2}=2\pi(2)=4\pi$$

4. (b) : Let $I=\int e^{-x}\operatorname{cosec} x(1+\cot x)dx$

Put $-x = t \Rightarrow dx = -dt$

$$\therefore\ I=-\int e^t(-\operatorname{cosec} t+\operatorname{cosec} t\cot t)\,dt$$

$= e^t \operatorname{cosec} t + C = -e^{-x}\operatorname{cosec}x + C$

5. (c) : Given $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=144$ and $|\vec{a}|=4$

Now, $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=|\vec{a}|^2\cdot|\vec{b}|^2$

$\Rightarrow\ 144=16\cdot|\vec{b}|^2 \Rightarrow |\vec{b}|^2=9\Rightarrow|\vec{b}|=3$

6. (c) : We have, $n(A) = 3$

So, total number of binary operations on $A = 3^{3^2} = 3^9$

7. (c) : We have, $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$

$= 2 \times 1 - 3 = 2 - 3 = -1 \quad [\because\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1]$

8. (c) : Let $|A| = \begin{vmatrix}\sin^2\alpha & \cos^2\alpha\\ \cos^2\alpha & \sin^2\alpha\end{vmatrix}=0$

$\Rightarrow\ \sin^4\alpha - \cos^4\alpha = 0$

$\Rightarrow\ (\sin^2\alpha - \cos^2\alpha)(\sin^2\alpha + \cos^2\alpha) = 0$

$\Rightarrow\ \sin^2\alpha - \cos^2\alpha = 0 \Rightarrow \sin^2\alpha = \cos^2\alpha \Rightarrow \tan^2\alpha = 1$

$\Rightarrow\ \tan\alpha = \pm 1 \Rightarrow \alpha = \frac{\pi}{4}, \frac{3\pi}{4} \in (0, \pi)$

9. (b): $P(X \le 1) = F(1) = 0.2$

$P(X \le 2) = P(X = 1) + P(X = 2)$

$\Rightarrow\ P(X = 2) = 0.37 - 0.2 = 0.17$

$P(X \le 3) = P(X = 1) + P(X = 2) + P(X = 3)$

$\Rightarrow\ P(X = 3) = 0.48 - 0.2 - 0.17 = 0.11$

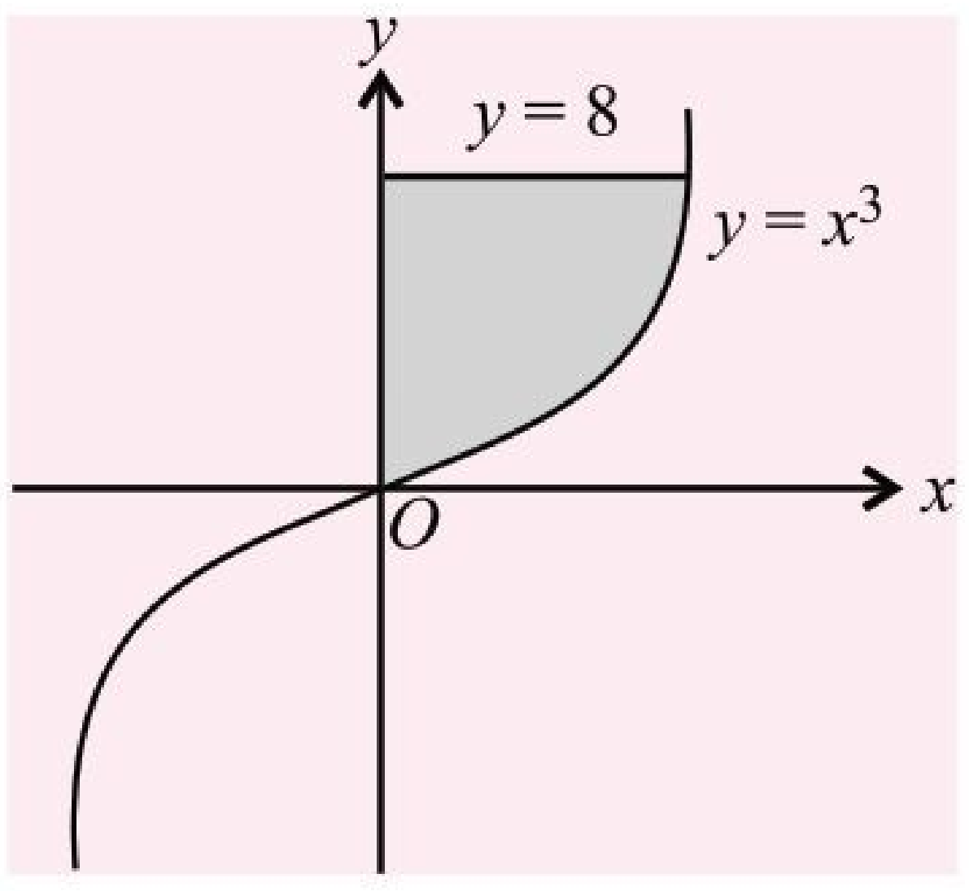
Similarly, $P(X = 4) = 0.14$, $P(X = 5) = 0.23$ and $P(X = 6) = 0.15$

X	1	2	3	4	5	6
$F(X)$	0.2	0.37	0.48	0.62	0.85	1
$P(X)$	0.2	0.17	0.11	0.14	0.23	0.15

$\therefore\ P(3 < X \le 5) = P(X = 4) + P(X = 5)$

$= 0.14 + 0.23 = 0.37$

10. (d): Given equation of curve is $y = x^3 \Rightarrow x = (y)^{1/3}$



$\therefore$ Required area

$$=\int_0^8 x\,dy=\int_0^8 y^{1/3}\,dy=\frac{3}{4}\left|y^{4/3}\right|_0^8$$

$$=\frac{3}{4}\left[(8)^{4/3}\right]=\frac{3}{4}\times 16=12 \text{ sq.units}$$

11. (b): Since $f(x)$ is continuous at $x = 0$.

$\therefore\ f(0) = \lim_{x\to 0} \log(\sec^2 x)^{\cot^2 x}$

$\Rightarrow\ K=\lim_{x\to 0}\cot^2 x\cdot\log(1+\tan^2 x)$

$\Rightarrow\ K=\lim_{x\to 0}\frac{\log(1+\tan^2 x)}{\tan^2 x}$

$\Rightarrow\ K=1 \qquad \left[\because\ \lim_{x\to 0}\frac{\log(1+x)}{x}=1\right]$

12. (d): As we know that $(A^{-1})^{-1} = A$

Here, $A^{-1}=\frac{1}{11}\begin{bmatrix}-3 & 4\\ 5 & -3\end{bmatrix}$

$\therefore\ A=\left\{\frac{1}{11}\begin{bmatrix}-3 & 4\\ 5 & -3\end{bmatrix}\right\}^{-1}=11\begin{bmatrix}-3 & 4\\ 5 & -3\end{bmatrix}^{-1}$

$\Rightarrow\ A = 11B^{-1}$ (say) ...(i)

Now, $B=\begin{bmatrix}-3 & 4\\ 5 & -3\end{bmatrix}$

$\therefore\ |B| = 9 - 20 = -11 \ne 0$

So, $\text{adj } B=\begin{bmatrix}-3 & -5\\ -4 & -3\end{bmatrix}'=\begin{bmatrix}-3 & -4\\ -5 & -3\end{bmatrix}$

So, $B^{-1}=\frac{-1}{11}\begin{bmatrix}-3 & -4\\ -5 & -3\end{bmatrix}=\frac{1}{11}\begin{bmatrix}3 & 4\\ 5 & 3\end{bmatrix}$

Now, from (i) we have $A=\begin{bmatrix}3 & 4\\ 5 & 3\end{bmatrix}$

13. (d): We have $f(x)=\sin^{-1}\left(\frac{x+5}{2}\right)$

$\Rightarrow\ -1\le\frac{x+5}{2}\le 1 \Rightarrow -2-5\le x\le 2-5$

$\Rightarrow\ -7 \le x \le -3$

$\therefore$ Domain of $f(x) = [-7, -3]$

14. (a): We have, $\begin{bmatrix}x+y & x-y\\ 2x+z & x+z\end{bmatrix}=\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}$

On comparing, we get

$x + y = 0$ ...(i)

$x - y = 0$ ...(ii)

$2x + z = 1$ ...(iii)

$x + z = 1$ ...(iv)

On solving (iii) and (iv), we get $x = 0$, $z = 1$.

Hence, from (i), we get $y = 0$

$\therefore\ x = 0, y = 0, z = 1$.

15. (c) : We have, $I_n=\int(\log x)^n dx$

$\Rightarrow\ I_n=(\log x)^n\cdot x-n\cdot I_{n-1}+C\Rightarrow I_n+nI_{n-1}=(\log x)^n\cdot x+C$

16. (b)

17. (a) : Given $f(x)=\begin{cases}x\sin\frac{1}{x}, & x\ne 0\\ k, & x=0\end{cases}$

Since, $f(x)$ is continuous at $x = 0$

$\therefore\ \lim_{x\to 0^-}f(x)=\lim_{x\to 0^+}f(x)=f(0)$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}\left[x\sin\frac{1}{x}\right]$$

Now, $-1\le \sin\left(\frac{1}{x}\right)\le 1 \;\Rightarrow -x\le x\sin\left(\frac{1}{x}\right)\le x$

$\Rightarrow\;\lim_{x\to 0}(-x)\le \lim_{x\to 0}\left(x\sin\frac{1}{x}\right)\le \lim_{x\to 0}(x)$

$\Rightarrow\;0\le \lim_{x\to 0}\left[x\sin\frac{1}{x}\right]\le 0\Rightarrow \lim_{x\to 0}\left[x\sin\frac{1}{x}\right]=0$ ...(i)

and $f(0) = k$ ...(ii)

∴ From (i) and (ii), we get $k = 0$

18. (a) : Let p be the perimeter and A be the area of the sector.

∴ $p = l + 2r = r(\theta + 2)$

Also, $A=\frac{1}{2}r^2\theta=\frac{1}{2}\frac{p^2}{(\theta+2)^2}\theta\Rightarrow\frac{dA}{d\theta}=0\Rightarrow\theta=2^\circ$

19. (a) : We have,

$\vec{a}+\vec{b}=(\hat{i}+\hat{j}+\hat{k})+(\hat{i}+3\hat{j}+5\hat{k})=2\hat{i}+4\hat{j}+6\hat{k}$

$\vec{b}+\vec{c}=(\hat{i}+3\hat{j}+5\hat{k})+(7\hat{i}+9\hat{j}+11\hat{k})=8\hat{i}+12\hat{j}+16\hat{k}$

Now, $(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 4 & 6\\ 8 & 12 & 16\end{vmatrix}$

$=\hat{i}(-8)-\hat{j}(-16)+\hat{k}(-8)\;=-8\hat{i}+16\hat{j}-8\hat{k}$

So, area of parallelogram $=\frac{1}{2}\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right|$

$=\frac{1}{2}\sqrt{(-8)^2+(16)^2+(-8)^2}=4\sqrt{6}$ sq. units

20. (c) : Here $f(x) = x^3 - 6x^2 + 9x + 10$

$\Rightarrow\; f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3)$

$= 3(x - 3)(x - 1)$

$f(x)$ will be increasing if $f'(x) \ge 0 \Rightarrow 3(x - 3)(x - 1) \ge 0$

$\Rightarrow\; x \le 1, x \ge 3$

∴ Function $f(x)$ increases when $x \in (-\infty, 1] \cup [3, \infty)$

21. (b): The given differential equation is

$x\frac{dy}{dx}+2y=x^2 \;\Rightarrow\; \frac{dy}{dx}+\frac{2}{x}y=x$

Here, I.F. = $e^{\int\frac{2}{x}dx} = e^{2\log x} = x^2$

∴ Required solution is,

$y\cdot x^2=\int x\cdot x^2dx+c \;\Rightarrow\; yx^2=\frac{x^4}{4}+c$

Given, $y(1) = 1 \;\Rightarrow\; c = \frac{3}{4}$.

Now, $yx^2 = \frac{x^4}{4}+\frac{3}{4}$

At $x = \frac{1}{2}$, $y\cdot\left(\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^4\cdot\frac{1}{4}+\frac{3}{4}=\frac{49}{64}\Rightarrow y=\frac{49}{16}$

Hence, $y\left(\frac{1}{2}\right)=\frac{49}{16}$

22. (b): Let $A(a, 0, 0)$, $B(0, b, 0)$ and $C(0, 0, c)$

∴ $(1, 2, 3)=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)\Rightarrow a=3,\ b=6,\ c=9$

∴ Equation of plane is $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

23. (a): $A=\begin{bmatrix}2 & 2\\ -3 & 2\end{bmatrix},\ B=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$|A| = 4 + 6 = 10 \ne 0$ and $|B| = 0 + 1 = 1 \ne 0$

∴ A^{-1}, B^{-1} exists.

$(B^{-1}A^{-1})^{-1} = (A^{-1})^{-1}(B^{-1})^{-1} = AB$

$=\begin{bmatrix}2 & 2\\ -3 & 2\end{bmatrix}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}0+2 & -2+0\\ 0+2 & 3+0\end{bmatrix}=\begin{bmatrix}2 & -2\\ 2 & 3\end{bmatrix}$

24. (a): We have, $\int_{-1}^{4}f(x)dx=4$

$\Rightarrow\;\int_{-1}^{2}f(x)\,dx+\int_{2}^{4}f(x)\,dx=4$ ...(i)

Also, $\int_{2}^{4}(3-f(x))dx=7\Rightarrow\int_{2}^{4}3dx-\int_{2}^{4}f(x)dx=7$

$\Rightarrow\;3[x]_2^4-7=\int_{2}^{4}f(x)dx\;\Rightarrow\;\int_{2}^{4}f(x)dx=-1$ ...(ii)

From (i) and (ii), we have

$\int_{-1}^{2}f(x)dx=4+1=5$

25. (b): Since, $n = 5$, $p = \frac{1}{3}$

∴ $q = 1 - p = 1 - \frac{1}{3}=\frac{2}{3}$

$\Rightarrow\;(2 < X < 4) = P(X = 3)$

$={}^5C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^2$

$=\frac{10\times 4}{3^5}=\frac{40}{243}$

26. (b) : We have,

$$10\tan(\cot^{-1}3+\cot^{-1}7)=10\tan\left(\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}\right)$$

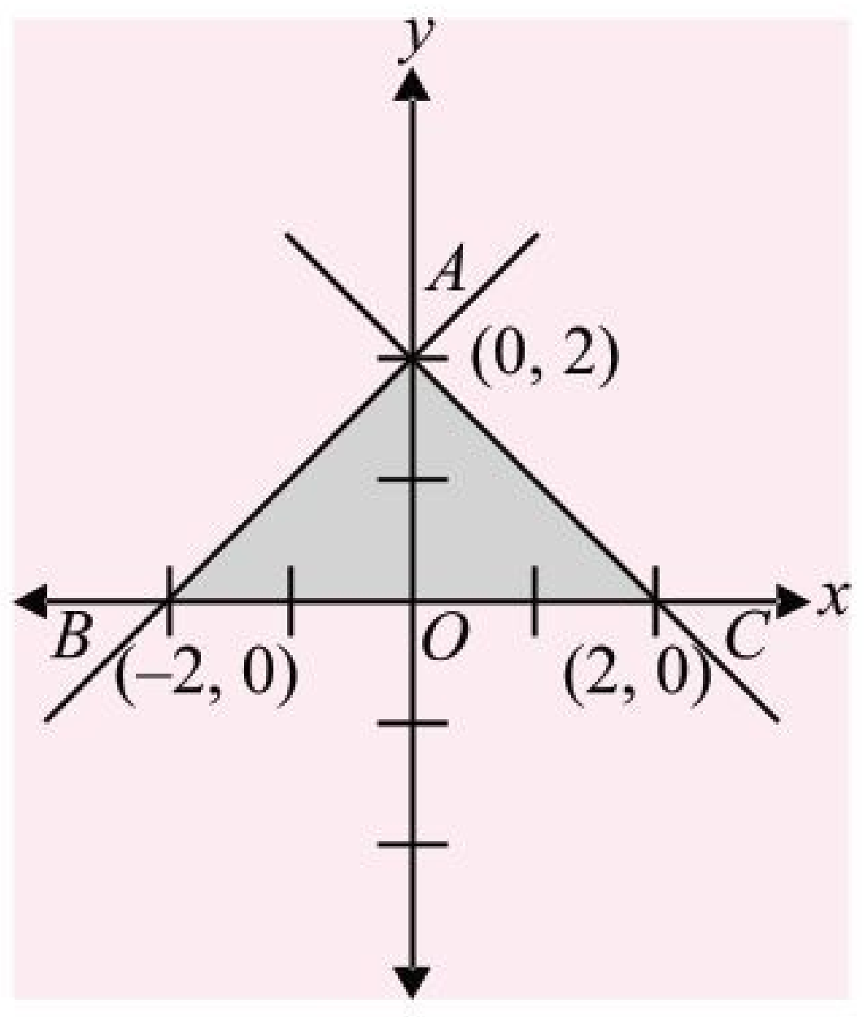
$$=10\tan\left[\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{3}\times\frac{1}{7}}\right)\right]=10\tan\left(\tan^{-1}\frac{1}{2}\right)$$

$$=10\times\frac{1}{2}=5$$

27. (c) : We have, $y = x + 2$ ...(1)

$y = 2 - x$ ...(2)

Point of intersection of two lines are (0, 2)



∴ Area bounded by lines

$$=\int_{-2}^{0}(x+2)dx+\int_{0}^{2}(2-x)dx$$

$$=\left[\frac{x^2}{2}+2x\right]_{-2}^{0}+\left[2x-\frac{x^2}{2}\right]_{0}^{2}$$

$$=0-(2-4)+(4-2)-0$$

= 4 square units

28. (a) : We have, $\tan^{-1}(x^2+y^2)=\alpha \Rightarrow x^2+y^2=\tan\alpha$

Differentiating w.r.t. x, we get

$$2x+2y\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{-x}{y}$$

29. (a) : $f'(x)=\begin{cases}-1 & -a<x<a\\ 3 & a\le x\end{cases}$

$f'(a^-) = -1$ and $f'(a^+) = 3$

∴ $f'(a^-)\neq f'(a^+)$

30. (a)

31. (b) : $4\otimes_7 x=5\Rightarrow x=3$

32. (d) : We have, $\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ 4\end{bmatrix}$

$\Rightarrow x+y=2$ ....(i) and $-x+y=4$ ....(ii)

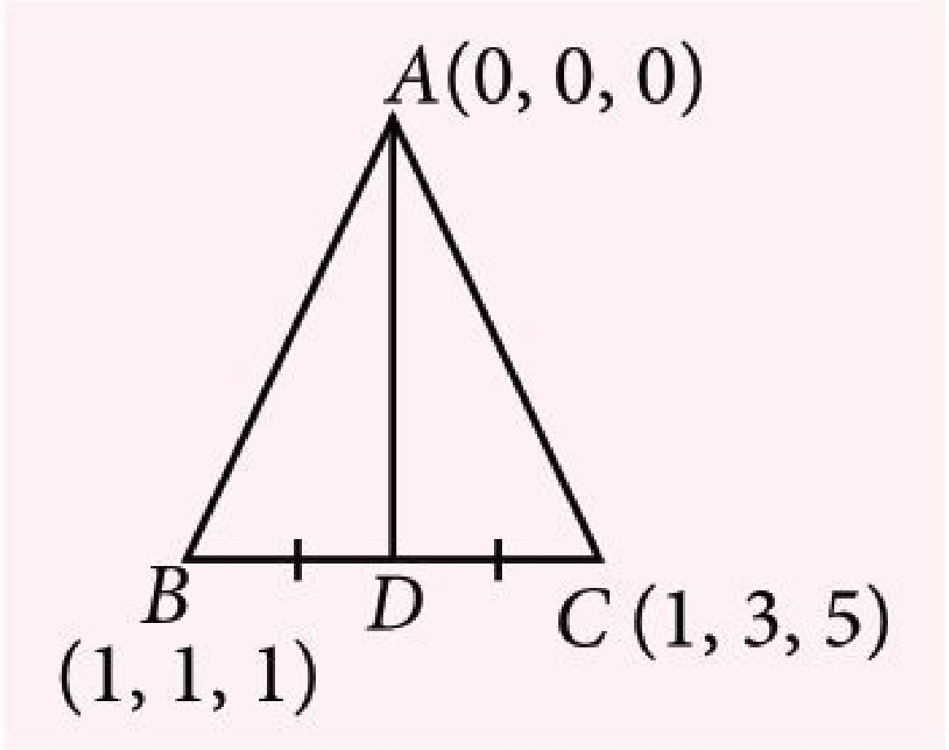
From (i) and (ii) we get, $x=-1$, $y=3$

33. (a) : $\displaystyle\int_0^{\pi/2}\frac{\tan^7 x}{\cot^7 x+\tan^7 x}dx=\frac{\pi}{4}$

$$\left[\because\int_0^{\pi/2}\frac{f(x)}{f(x)+f\left(\frac{\pi}{2}-x\right)}dx=\frac{\pi}{4}\right]$$

34. (a)

35. (c) : Let the position vectors of A, B and C are $0\hat{i}+0\hat{j}+0\hat{k}$, $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}+5\hat{k}$



∵ AD is the median of ΔABC.

∴ D is the midpoint of BC.

⇒ Position vector of D

$$=\left(\frac{(\hat{i}+\hat{j}+\hat{k})+(\hat{i}+3\hat{j}+5\hat{k})}{2}\right)$$

$$=\hat{i}+2\hat{j}+3\hat{k}$$

$$\therefore\ \overrightarrow{AD}=(1-0)\hat{i}+(2-0)\hat{j}+(3-0)\hat{k}=\hat{i}+2\hat{j}+3\hat{k}$$

$$\Rightarrow |\overrightarrow{AD}|=\sqrt{1^2+2^2+3^2}=\sqrt{14}$$

36. (d) : We have, $\dfrac{(p\hat{i}-\hat{j}+2\hat{k})\cdot(2\hat{i}-p\hat{j}-\hat{k})}{\sqrt{p^2+1+4}\sqrt{4+p^2+1}}=\cos\dfrac{\pi}{3}$

$$\Rightarrow \frac{2p+p-2}{\sqrt{p^2+5}\sqrt{p^2+5}}=\cos\frac{\pi}{3}\Rightarrow\frac{3p-2}{p^2+5}=\frac{1}{2}$$

$\Rightarrow 6p-4=p^2+5\Rightarrow p^2-6p+9=0$

$\Rightarrow (p-3)^2=0\Rightarrow p=3$

37. (a) : If A and B are independent, then A' and B' are also independent

$\Rightarrow P(A'\cap B')=P(A')P(B')=(1-P(A))(1-P(B))$

38. (a)

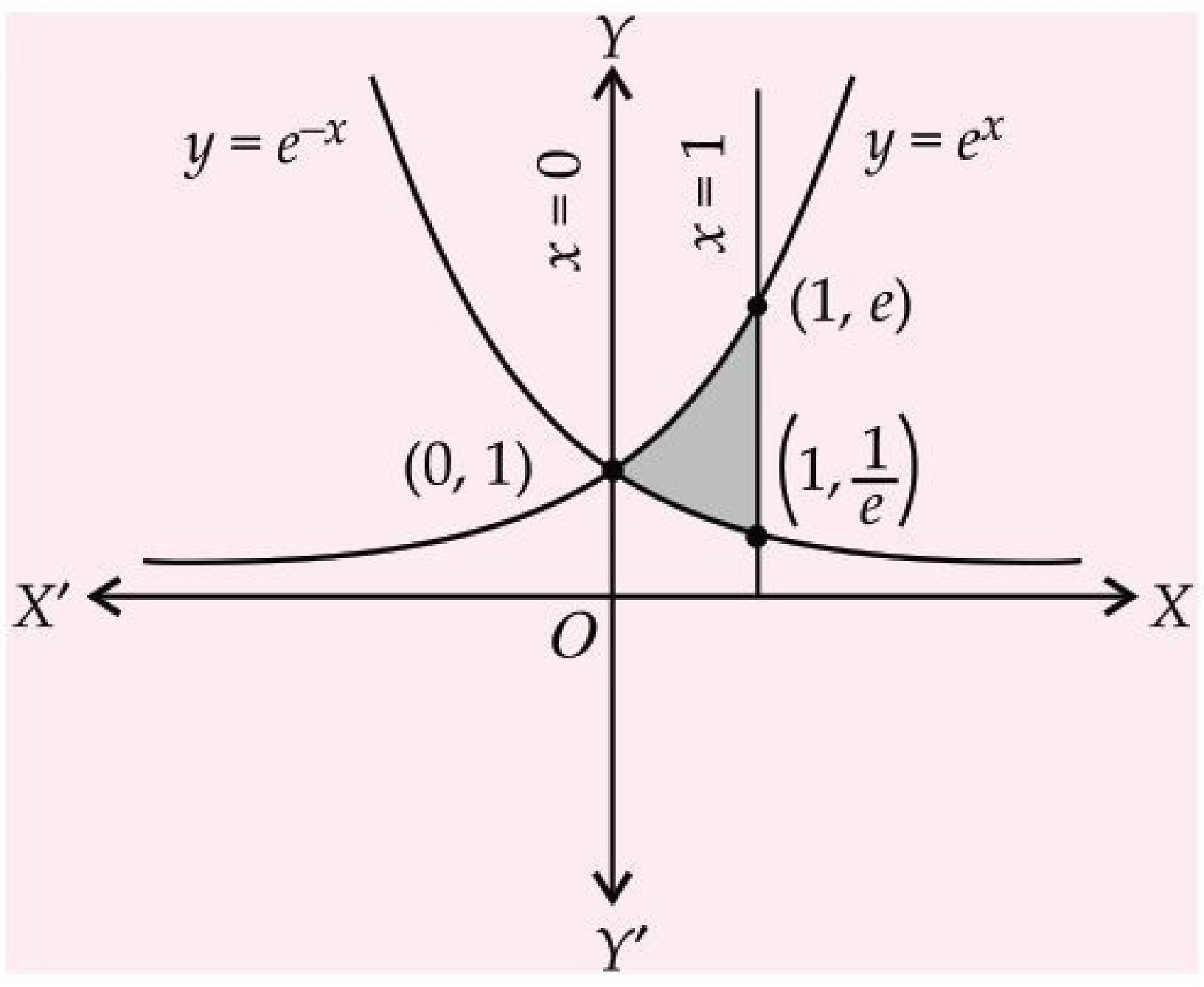
39. (a) : $\cos^{-1}\left(\cos\left(\dfrac{7\pi}{5}\right)\right)=\cos^{-1}\left(\cos\left(2\pi-\dfrac{3\pi}{5}\right)\right)$

$$=\cos^{-1}\left(\cos\left(\frac{3\pi}{5}\right)\right)=\frac{3\pi}{5}$$

40. (d) : The given differential equation can be rewritten as, $\left(1+3\dfrac{dy}{dx}\right)^2=64\left(\dfrac{d^3y}{dx^3}\right)^3$.

Thus, order is 3 and degree is 3.

41. (c) :



∴ Required area = Area of shaded region in the figure

$$=\int_0^1(e^x-e^{-x})dx=\left[e^x+e^{-x}\right]_0^1$$

$= e + e^{-1} - e^0 - e^0 = (e + e^{-1} - 2)$ sq. units

42. (d): We have, $I_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$

$$= \int_0^{\frac{\pi}{4}} (\tan x)^{n-2} \cdot \tan^2 x\, dx \;= \int_0^{\frac{\pi}{4}} (\tan x)^{n-2} (\sec^2 x - 1)\, dx$$

$$= \int_0^{\frac{\pi}{4}} (\tan x)^{n-2} \sec^2 x\, dx - \int_0^{\frac{\pi}{4}} (\tan x)^{n-2}\, dx$$

$$= \left[\frac{(\tan x)^{n-1}}{n-1}\right]_0^{\frac{\pi}{4}} - I_{n-2} \quad \Rightarrow I_n + I_{n-2} = \frac{1}{n-1}$$

$$\therefore I_{10} + I_{10-2} = \frac{1}{10-1} \quad \Rightarrow I_{10} + I_8 = \frac{1}{9}$$

43. (b): We have, $f(x) = 3x + 5$, $g(x) = x^2 - 1$

$\therefore$ $(fog)(x^2 - 1) = f[g(x^2 - 1)] = f[(x^2 - 1)^2 - 1]$
$= f(x^4 - 2x^2) = 3(x^4 - 2x^2) + 5 = 3x^4 - 6x^2 + 5$

44. (c) : According to question, we have,

$\frac{dV}{dt} = 10,$ where V is the volume of spherical balloon

$$\Rightarrow \frac{d\left(\frac{4}{3}\pi r^3\right)}{dt} = 10 \Rightarrow \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt} = 10 \Rightarrow \frac{dr}{dt} = \frac{5}{2\pi r^2}$$

At $r = 15$ cm, $\frac{dr}{dt} = \frac{5}{2\pi \times 15 \times 15} = \frac{1}{90\pi}$ cm/sec

45. (a) : We have, $y = x^3 - 6x + 5$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = 3x^2 - 6$$

$\therefore$ Slope of tangent $= \left.\frac{dy}{dx}\right|_{(2,1)} = 12 - 6 = 6$

So, required equation of tangent is

$y - 1 = 6(x - 2) \Rightarrow 6x - y - 11 = 0$

46. (d): (i) It is given that, $A \subset B$

$\Rightarrow A \cap B = A$

$$\therefore \quad P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

(ii) If $A \cap B = \phi$

Then $P(B|A) = \frac{P(A \cap B)}{P(A)} = 0$

47. (d): Let the equation of plane be

$ax + by + cz + d = 0$ ...(1)

Now, the given line is $\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{3}$ ...(2)

Also, plane (1) is perpendicular to the line (2).

So, $\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda$ (say) $\Rightarrow a = \lambda, b = 2\lambda, c = 3\lambda$ ...(3)

Also, plane (1) passes through (2, 3, 4), so, we get

$2a + 3b + 4c + d = 0$ ...(4)

From (3) and (4), we have $2\lambda + 6\lambda + 12\lambda + d = 0$

$\Rightarrow d = -20\lambda$

$\therefore$ Required equation of plane is,

$\lambda x + 2\lambda y + 3\lambda z - 20\lambda = 0 \Rightarrow x + 2y + 3z = 20$

48. (a) : We have, $\frac{x\frac{dy}{dx} - y}{\sqrt{x^2 - y^2}} = 10x^2$

$$\Rightarrow \frac{x\,dy - y\,dx}{x^2} = 10x \cdot \sqrt{1 - \frac{y^2}{x^2}}\, dx \quad \text{...(i)}$$

Put $\frac{y}{x} = t \Rightarrow \frac{x\,dy - y\,dx}{x^2} = dt$

$\therefore$ Equation (i) becomes, $\frac{dt}{\sqrt{1-t^2}} = 10x\,dx$

Integrating both sides, we get $\int \frac{dt}{\sqrt{1-t^2}} = 10\int x\,dx$

$$\Rightarrow \sin^{-1} t = \frac{10x^2}{2} + C \Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = 5x^2 + C$$

49. (d): We have,

$$\sin[2\sin^{-1} 0.8] = \sin\left[\sin^{-1}\left(2 \times 0.8\sqrt{1 - (0.8)^2}\right)\right]$$

$= 2 \times 0.8 \times 0.6 = 0.96$

50. (a) : Let $A = \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix}$ $\therefore A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ [Given]

Now $AA^{-1} = I \Rightarrow \begin{bmatrix} 1 & 5 \\ 7 & -3 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a + 5c & b + 5d \\ 7a - 3c & 7b - 3d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now $b + 5d = 0$ ...(i) and $7b - 3d = 1$ ...(ii)

Solving (i) and (ii), we get $d = \frac{-1}{38}$.

◆ ◆

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(a)	**2.**	(d)	**3.**	(a)	**4.**	(d)	**5.**	(d)
6.	(b)	**7.**	(a,c)	**8.**	(a,c)	**9.**	(a,b,c)	**10.**	(a,d)
11.	(b,c)	**12.**	(a)	**13.**	(b)	**14.**	(a)	**15.**	(c)
16.	(a)	**17.**	(1)	**18.**	(6)	**19.**	(5)	**20.**	(5)

YOU ASK
WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Let $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ be two roots of $(x+1)^n + x^n + 1 = 0$ where α, β are roots of $x^2 + px + q = 0$. If α, β are also roots of the equation $x^{2n} + p^n x^n + q^n = 0$, then show that n must be an even integer when $n \neq 0$.

(Garima, Maharashtra)

Ans. As $\frac{\alpha}{\beta}$ is a root of $(x+1)^n + x^n + 1 = 0$,

$$\left(\frac{\alpha}{\beta}+1\right)^n + \left(\frac{\alpha}{\beta}\right)^n + 1 = 0$$

$$\Rightarrow \quad (\alpha+\beta)^n + \alpha^n + \beta^n = 0. \qquad ...(1)$$

But α, β are roots of $x^2 + px + q = 0$

$\therefore \quad \alpha + \beta = -p, \alpha\beta = q$

$\therefore \quad$ From (1), $(-p)^n + \alpha^n + \beta^n = 0 \qquad ...(2)$

As α, β are roots of $x^{2n} + p^n x^n + q^n = 0$, we get

$$\alpha^{2n} + p^n \alpha^n + q^n = 0$$

and $\beta^{2n} + p^n \beta^n + q^n = 0$.

Subtracting the above two eqns, we get

$\alpha^{2n} - \beta^{2n} + p^n(\alpha^n - \beta^n) = 0$

or $\quad (\alpha^n + \beta^n)(\alpha^n - \beta^n) + p^n(\alpha^n - \beta^n) = 0$

$\therefore \quad \alpha^n + \beta^n + p^n = 0 \qquad ...(3)$

$[\because \alpha^n \neq \beta^n \text{ as } \alpha \neq \beta]$

From $(2) - (3)$, we get

$(-p)^n - p^n = 0$

$\therefore \quad (-p)^n = p^n \quad \Rightarrow \quad n$ is an even integer.

2. Prove that

$(b - a)\sec^2 a < \tan b - \tan a < (b - a)\sec^2 b,$

where $0 < a < b < \frac{\pi}{2}$. *(Arunav Biswas, Warangal)*

Ans. Consider the function $f(x) = \tan x \ \forall \ x \in (a, b)$.

Clearly, $f(x)$ is continuous and differentiable in $\left(0, \frac{\pi}{2}\right)$ and so in (a, b).

Therefore, by Lagrange's theorem

$$\frac{f(b) - f(a)}{b - a} = f'(c), \text{ where } a < c < b$$

$$\Rightarrow \quad \frac{\tan b - \tan a}{b - a} = \sec^2 c$$

or, $\tan b - \tan a = (b - a)\sec^2 c \qquad ...(1)$

But $a < c < b \quad \Rightarrow \quad \sec a < \sec c < \sec b$

$\therefore \quad (b - a)\sec^2 a < (b - a)\sec^2 c < (b - a)\sec^2 b$

$\Rightarrow \quad (b - a)\sec^2 a < \tan b - \tan a < (b - a)\sec^2 b.$

(From (1))

3. Find the locus of the point from which the two tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are inclined at an angle θ. *(Rakesh Gupta, Punjab)*

Ans. Equation of a tangent to the given ellipse can be written as

$$y - mx = \sqrt{a^2m^2 + b^2} \qquad ...(1)$$

If $P(h, k)$ be a point lying on the (1), then, we have

$$k - mh = \sqrt{a^2m^2 + b^2}$$

i.e., $(h^2 - a^2)m^2 - 2hkm + k^2 - b^2 = 0 \qquad ...(2)$

Let the slope of two tangents or roots of equation (2) be m_1 and m_2.

Then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

i.e., $(1 + m_1m_2)^2\tan^2\theta = (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$$\Rightarrow \quad \left(1 + \frac{k^2 - b^2}{h^2 - a^2}\right)^2 \tan^2\theta = \frac{4h^2k^2}{(h^2 - a^2)^2} - \frac{4(k^2 - b^2)}{(h^2 - a^2)}$$

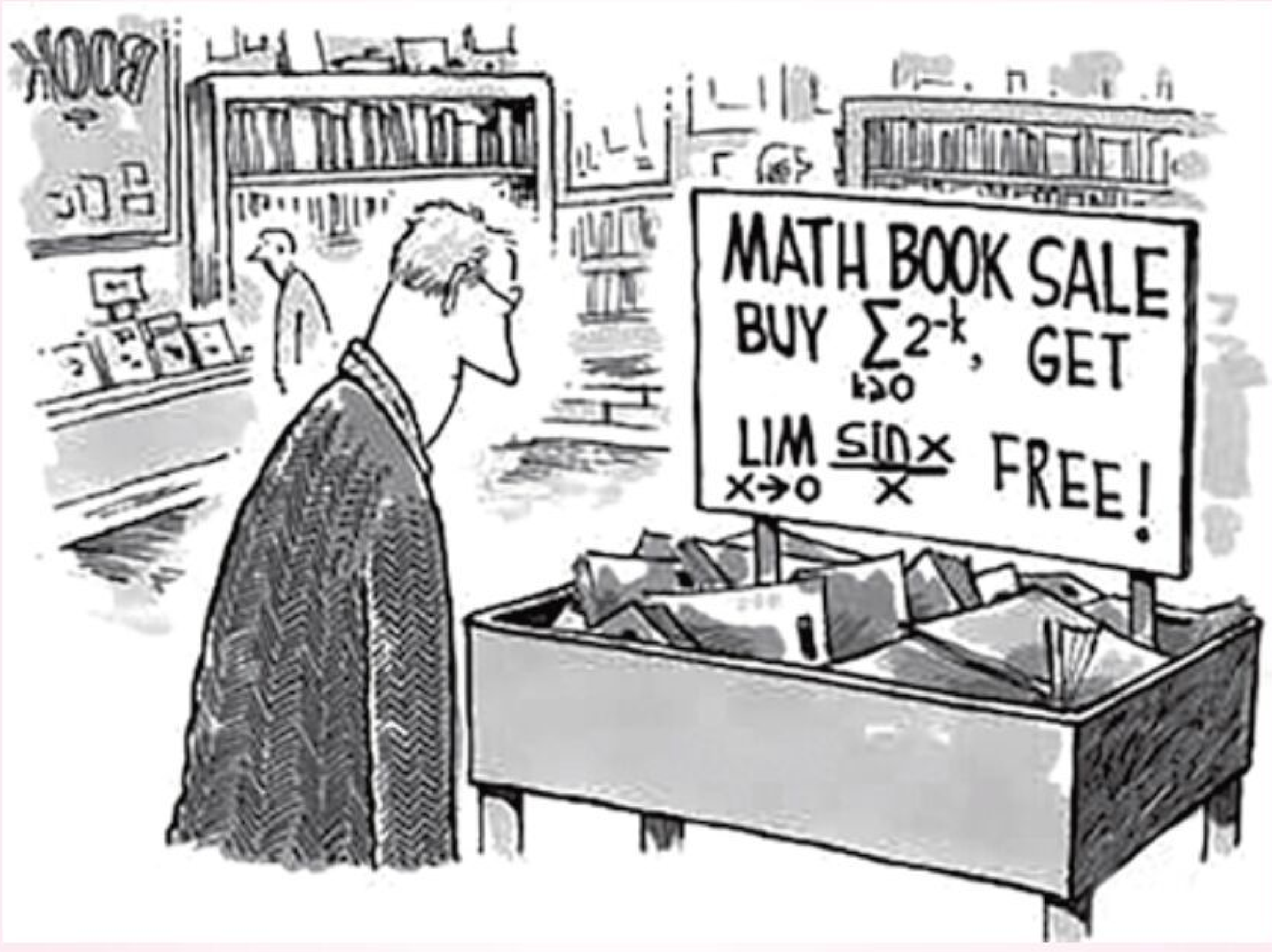
i.e., $(h^2 + k^2 - a^2 - b^2)^2\tan^2\theta = 4(b^2h^2 + a^2k^2 - a^2b^2)$

$\therefore \quad$ The required locus is

$(x^2 + y^2 - a^2 - b^2)^2 \tan^2\theta = 4(b^2x^2 + a^2y^2 - a^2b^2)$.

◆

COMIC CAPSULE


BOOK
MATH BOOK SALE
BUY $\sum_{k\geq 0} 2^{-k}$, GET
$\lim_{x\to 0} \frac{\sin x}{x}$ FREE!

OLYMPIAD CORNER

1. Let p be a prime. Find all solutions in positive integers of the equation : $\frac{2}{a}+\frac{3}{b}=\frac{5}{p}$.
2. Find all triples (x,y,z) of natural numbers such that y is a prime number, y and 3 do not divide z, and $x^3-y^3=z^2$.
3. Solve the simultaneous equations $x^2(y+z)=1,\ y^2(z+x)=8,\ z^2(x+y)=13$
4. In a triangle ABC, and A is twice angle B. Show that $a^2=b(b+c)$.
5. Find an integer $n>1$ so that there exist n consecutive integer squares having an average of n^2.
6. Find all nonzero reals $x>-1$ which satisfy $$\frac{x^2}{\left(x+1-\sqrt{x+1}\right)^2}<\frac{x^2+3x+18}{(x+1)^2}.$$
7. For non-negative reals x, y and z satisfying $x+y+z=1$, show that $\left(\frac{1}{x}+1\right)\left(\frac{1}{y}+1\right)\left(\frac{1}{z}+1\right)\geq 64.$
8. In a television commercial some months ago, a pizza restaurant announced a special sale on two pizzas, in which each pizza could independently contain up to five of the toppings the restaurant had available (no topping at all is also an option). In the commercial, a small boy declared that there were a total of 1048576 different possibilities for the two pizzas one could order. How many toppings are available at the restaurant?

SOLUTIONS

1. We have, $p(3a+2b)=5ab$. Hence, there are three cases to consider.

First case : $p=5$. Then, we get

$3a+2b=ab$ or $(a-2)(b-3)=6=1\cdot 6=2\cdot 3$

Hence, all pairs of positive integers (a, b) are $(3, 9)$, $(4, 6)$, $(5, 5)$ and $(8, 4)$.

Second case : p divides a.

Let $a=a_1p$.

Hence, $3a_1p=b(5a_1-2)$.

Then, p divides either b or $5a_1-2$.

If $b=b_1p$, then $3a_1=b_1(5a_1-2)$

or $(5a_1-2)(5b_1-3)=6$

which has only one solution $(a_1, b_1)=(1, 1)$, when $(a, b)=(p, p)$. Otherwise, $5a_1-2=a_2p$. Therefore,

$$3\frac{a_2p+2}{5}=ba_2 \text{ or } 3a_2p+6=5ba_2.$$

Hence, a_2 divides 6.

If $a_2=1$, then $(a, b)=\left(\frac{p(p+2)}{5},\frac{3(p+2)}{5}\right)$, but only if $p\equiv 3 \pmod 5$.

If $a_2=2$, then $(a, b)=\left(\frac{2p(p+1)}{5},\frac{3(p+1)}{5}\right)$, but only if $p\equiv 4 \pmod 5$.

If $a_2=3$, then $(a, b)=\left(\frac{p(3p+2)}{5},\frac{3p+2}{5}\right)$, but only if $p\equiv 1 \pmod 5$.

If $a_2=6$, then $(a, b)=\left(\frac{2p(3p+1)}{5},\frac{3p+1}{5}\right)$, but only if $p\equiv 3 \pmod 5$.

Third case : p divides b.

Let $b=b_1p$.

Hence, $2b_1p=a(5b_1-3)$

Then p divides either a or $5b_1-3$. If p divides a, then we have the same case (p divides a and b) as already considered above. Again (p, p) is the (only) solution. Otherwise, $5b_1-3=b_2p$. Therefore,

$$2\frac{b_2p+3}{5}=ab_2 \text{ or } 2b_2p+6=5ab_2.$$

If $b_2=1$, then $(a, b)=\left(\frac{2(p+3)}{5},\frac{p(p+3)}{5}\right)$, but only if $p\equiv 2 \pmod 5$.

If $b_2=2$, then $(a, b)=\left(\frac{2p+3}{5},\frac{p(2p+3)}{5}\right)$, but only if $p\equiv 1 \pmod 5$.

If $b_2 = 3$, then $(a, b) = \left(\frac{2(p+1)}{5}, \frac{3p(p+1)}{5}\right)$, but only if $p \equiv 4 \pmod 5$.

If $b_2 = 6$, then $(a, b) = \left(\frac{2p+1}{5}, \frac{3p(2p+1)}{5}\right)$, but only if $p \equiv 2 \pmod 5$.

[Note that if $p = 2$ or 3, this generates only two distinct solutions : (2, 2), (1, 6) for $p = 2$ and (3,3), (12, 2) for $p = 3$. If $p > 5$, then the three solutions are all distinct.]

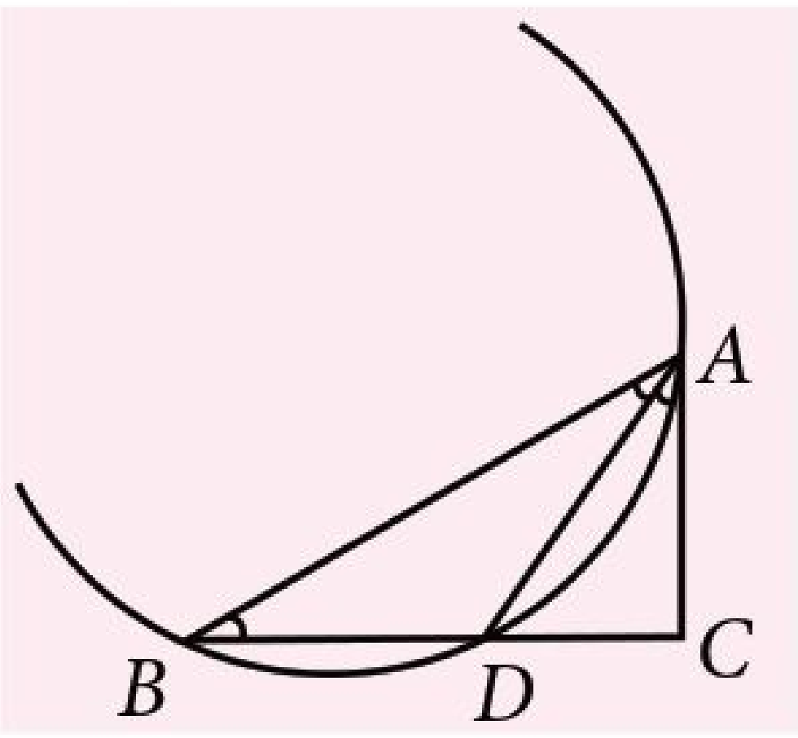
2. We rewrite the equation in the form
$(x - y)(x^2 + xy + y^2) = z^2$
Any common divisor of $x - y$ and $x^2 + xy + y^2$ also divides both z^2 and $(x^2 + xy + y^2) - (x + 2y)(x - y) = 3y^2$. Because z^2 and $3y^2$ are relatively prime by assumption, $x - y$ and $x^2 + xy + y^2$ must be relatively prime as well. Therefore, both $x - y$ and $x^2 + xy + y^2$ are perfect squares.

Writing $a = \sqrt{x-y}$, we have
$x^2 + xy + y^2 = (a^2 + y)^2 + (a^2 + y)y + y^2$
$= a^4 + 3a^2y + 3y^2$
and $4(x^2 + xy + y^2) = (2a^2 + 3y)^2 + 3y^2$.

Writing $m = 2\sqrt{x^2+xy+y^2}$ and $n = 2a^2 + 3y$,
we have $m^2 = n^2 + 3y^2$ or $(m - n)(m + n) = 3y^2$,
so $(m - n, m + n) = (1, 3y^2), (y, 3y)$, or $(3, y^2)$
In the first case, $2n = 3y^2 - 1$
and $4a^2 = 2n - 6y = 3y^2 - 6y - 1$.
Hence, $a^2 \equiv 2 \pmod 3$, which is impossible.
In the second case, $n = y < 2a^2 + 3y = n$, a contradiction.
In the third case, we have
$$4a^2 = 2n - 6y = y^2 - 6y - 3 < (y - 3)^2 .$$
When $y \ge 10$ we have $y^2 - 6y - 3 > (y - 4)^2$. Hence, we must actually have $y = 2,3,5$, or 7. In this case we have $a = \frac{\sqrt{y^2-6y-3}}{2}$, which is real only when $y = 7$, $a = 1$, $x = y + a^2 = 8$, and $z = 13$. This yields the unique solution $(x, y, z) = (8, 7, 13)$.

3. More generally we can replace the constants 1, 8, 13 by a^3, b^3, c^3 respectively. Then, by addition of the three equations and by multiplication of the three equations, we respectively get
$\sum x^2y = a^3 + b^3 + c^3$,
$(xyz)^2\,[2xyz + \sum x^2y] = (abc)^3$,
where the sums are symmetric over x, y, z. Hence,
$$2t^3 + t^2(a^3 + b^3 + c^3) = (abc)^3 \qquad \dots(1)$$
where, $t = xyz$. In terms of t, the original equations can be rewritten as
$$\frac{a^3}{tx} - \frac{1}{y} - \frac{1}{z} = 0, \quad \frac{-1}{x} + \frac{b^3}{ty} - \frac{1}{z} = 0, \quad \frac{-1}{x} - \frac{1}{y} + \frac{c^3}{tz} = 0$$
These latter homogeneous equations are consistent since, the eliminant is equation (1). Solving the last two equations for y and z, we get
$$y = \frac{x(b_1c_1 - 1)}{c_1 + 1}, \quad z = \frac{x(b_1c_1 - 2)}{b_1 + 1}$$
where, $b_1 = b^3/t$, $c_1 = c^3/t$. On substituting back in $x^2(y + z) = a^3$, we obtain x^3 and then x, y, z.

4. **1st solution:** Let AD be the internal bisector of $\angle A$, meeting BC at D. Then, $\frac{BD}{DC} = \frac{AB}{AC}$; adding 1 to both sides



we get $\frac{BD}{DC} = \frac{AB+AC}{AC}$ ...(i)
Since, $\angle CAD = \angle ABD$, AC is tangent to the circumcircle of ΔABD. Therefore, $BC.DC = AC^2$. ...(ii)
Now, (i) × (ii) $\Rightarrow BC^2 = AC(AB + AC)$.

2nd solution: $A = 2B \Rightarrow C = \pi - 3B$ $\sin C = \sin 3B$. Therefore,
$\sin^2 A = \sin^2 2B = 2\sin B \cos B \sin 2B$
$= \sin B(\sin B + \sin 3B) = \sin B(\sin B + \sin C)$
$\Rightarrow \quad a^2 = b(b + c)$.

5. We seek positive integers k and n, with $n > 1$, such that
$$n^2 = \frac{1}{2}\sum_{i=1}^{n}(k+i)^2 = \frac{1}{n}\left(\sum_{i=1}^{n}k^2 + 2k\sum_{i=1}^{n}i + \sum_{i=1}^{n}i^2\right)$$
$$= \frac{1}{n}\left(nk^2 + kn(n+1) + \frac{n}{6}(n+1)(2n+1)\right)$$
$$= k^2 + (n + 1)k + \frac{(n+1)(2n+1)}{6} \qquad \dots(i)$$
Note that n is odd [since otherwise the fraction in (i) is not an integer]
Complete the square in (i) by putting
$k = a - \frac{n+1}{2}$,
where k is an integer, then (i) becomes
$$n^2 = a^2 - \left(\frac{n+1}{2}\right)^2 + \frac{(n+1)(2n+1)}{6}$$
$$= a^2 + \frac{n+1}{12}[-3(n + 1) + 2(2n + 1)]$$
$$= a^2 + \frac{(n+1)(n-1)}{12} = a^2 + \frac{n^2-1}{12},$$
or $12a^2 = 11n^2 + 1$. Putting $n = a + t$ this becomes
$12a^2 = 11a^2 + 22at + 11t^2 + 1$
or $(a - 11t)^2 - 132t^2 = 1$.
With $x = a - 11t$ and $y = 2t$ one gets the Pell equation
$x^2 - 33y^2 = 1$. ...(ii)

The smallest positive integer solution is $y = 4$, $x = 23$, which implies $t = 2$, $a = 45$, $n = 47$, $k = 21$, and thus a solution to the problem is

$$\frac{22^2+23^2+\ldots+68^2}{47} = 47^2$$

From the theory of Pell equations (and putting $x_1 = 23$, $y_1 = 4$), there are infinitely many positive integer solutions (x_i, y_i) of (ii), given by

$x_i + y_i\sqrt{33} = (23+4\sqrt{33})^i$,

or alternately by

$x_{i+1} = 23x_i + 4\cdot 33y_i,\ \ y_{i+1} = 23y_i + 4x_i$.

The first four solutions and the corresponding values of k and n are

x	y	k	n
23	4	21	47
1057	184	988	2161
48599	8460	45449	99359
2234497	388976	2089688	4568353

6. Make the substitution $y=\sqrt{x+1}$, so $y \in (0, 1) \cup (1, \infty)$ and $x = y^2 - 1$. Then, the inequality is equivalent to

$$\frac{(y^2-1)^2}{(y^2-y)^2} < \frac{(y^2-1)^2+3(y^2-1)+18}{y^4}$$

$$\Leftrightarrow \quad \frac{(y+1)^2}{y^2} < \frac{y^4+y^2+16}{y^4}$$

$\Leftrightarrow (y + 1)^2y^2 < y^4 + y^2 + 16 \Leftrightarrow 2y^3 < 16$, so the condition is satisfied exactly when $y < 2$; *i.e.*, exactly when $y \in (0, 1) \cup (1, 2)$ and $x \in (-1, 0) \cup (0, 3)$.

7. Expanding, we first must show

$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz} \ge 64.$$

By AM-GM inequality,

$$xyz \le \left(\frac{x+y+z}{3}\right)^3 = \frac{1}{27} \text{ so that } \frac{1}{xyz} \ge 27.$$

Therefore,

$$1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{1}{xy}+\frac{1}{xz}+\frac{1}{yz}+\frac{1}{xyz}$$

$$\ge 1+\frac{3}{\sqrt[3]{xyz}}+\frac{3}{\sqrt[3]{x^2y^2z^2}}+\frac{1}{xyz}$$

$$=\left(1+\frac{1}{\sqrt[3]{xyz}}\right)^3 \ge 4^3 = 64.$$

8. If there are n toppings available at the restaurant then there are

$$k = \binom{n}{5}+\binom{n}{4}+\binom{n}{3}+\binom{n}{2}+\binom{n}{1}+\binom{n}{0}$$

ways to have a single pizza. Since, you could have up to five toppings (no toppings is a valid, although boring, choice). For two pizzas, you could have two of the same pizza (k ways) or two different pizzas ($\binom{k}{2} = k(k - 1)/2$ ways). Since, the total number of possible combinations of the two is given as 1048576, the following must be true:

$$\frac{k(k-1)}{2}+k=1048576$$

But this equation doesn't have any positive integer solutions. It would seem that the restaurant in question considers getting a pizza with bacon and cheese and a pizza with mushrooms, cheese and green pepper different from getting a pizza with mushrooms, cheese and green pepper and a pizza with bacon and cheese; *i.e.*, the order in which the order is placed matters. This seems silly to me, but so be it. In this case the equation that must be solved is $k^2 = 1048576 = 2^{20}$, so $k = 1024 = 2^{10}$. Now to find n we must solve

$$\binom{n}{5}+\binom{n}{4}+\binom{n}{3}+\binom{n}{2}+\binom{n}{1}+\binom{n}{0} = 2^{10}. \qquad \text{...(1)}$$

By the symmetry of Pascal's triangle we get

$$\binom{n}{0}=\binom{n}{n},\ \binom{n}{1}=\binom{n}{n-1},\ \ldots,\ \binom{n}{5}=\binom{n}{n-5},$$

so our equation (1) can also be written as

$$\binom{n}{n}+\binom{n}{n-1}+\binom{n}{n-2}+\binom{n}{n-3}+\binom{n}{n-4}+\binom{n}{n-5}=2^{10}.$$

If we add these two together we get

$$\binom{n}{n}+\binom{n}{n-1}+\ldots+\binom{n}{n-5}+\binom{n}{5}+\binom{n}{4}+\ldots+\binom{n}{0}$$

$= 2 \cdot 2^{10} = 2^{11}$,

which is satisfied if $n = 11$, since

$$\sum_{i=0}^{n}\binom{n}{i} = 2^n.$$

Thus, the restaurant has 11 different toppings.

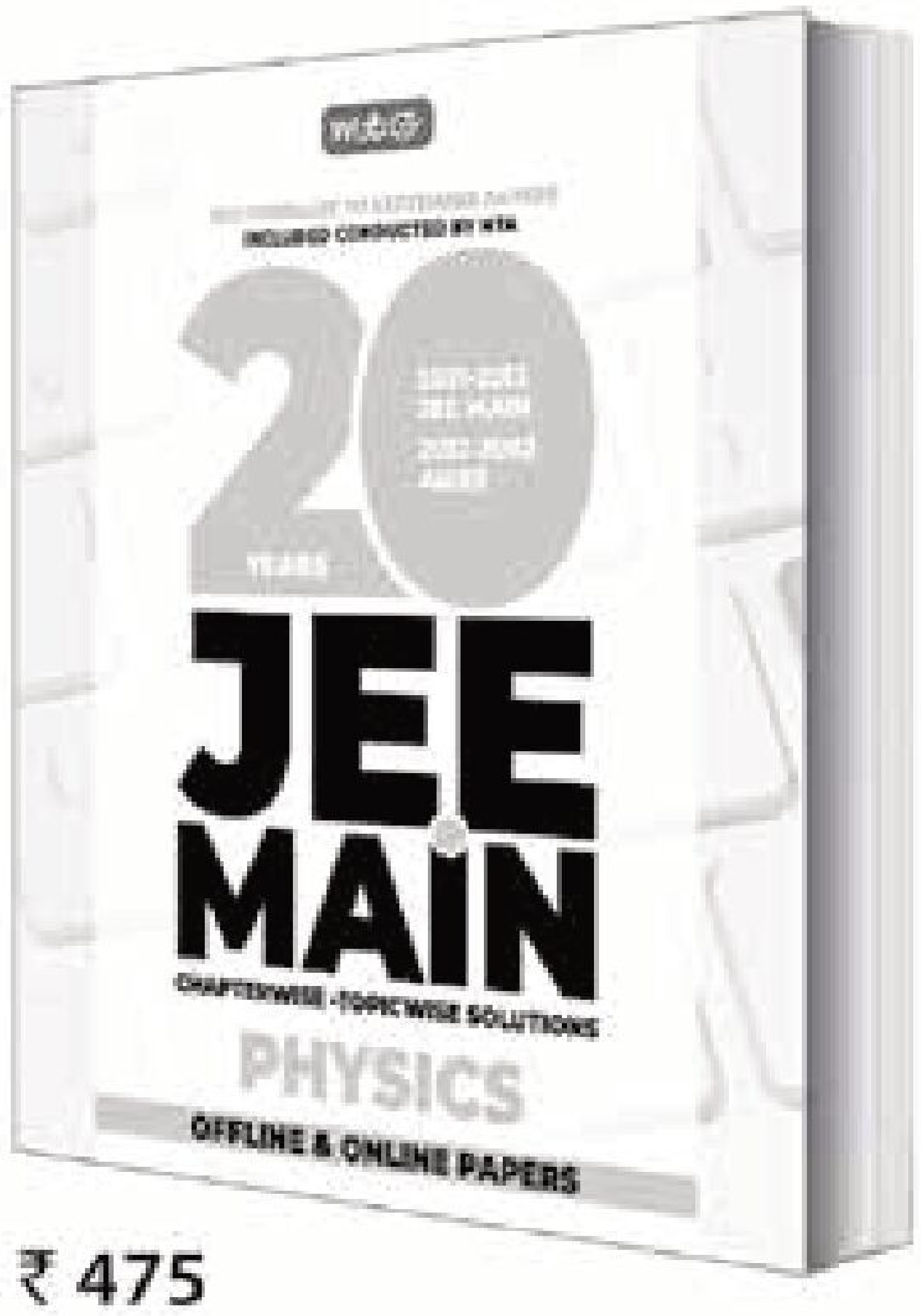
If the order in which the order is placed doesn't matter, then the number of two-pizza orders is

$$\frac{k(k+1)}{2}=\frac{1024\cdot 1025}{2}=524800, \text{ not } 1048576.$$

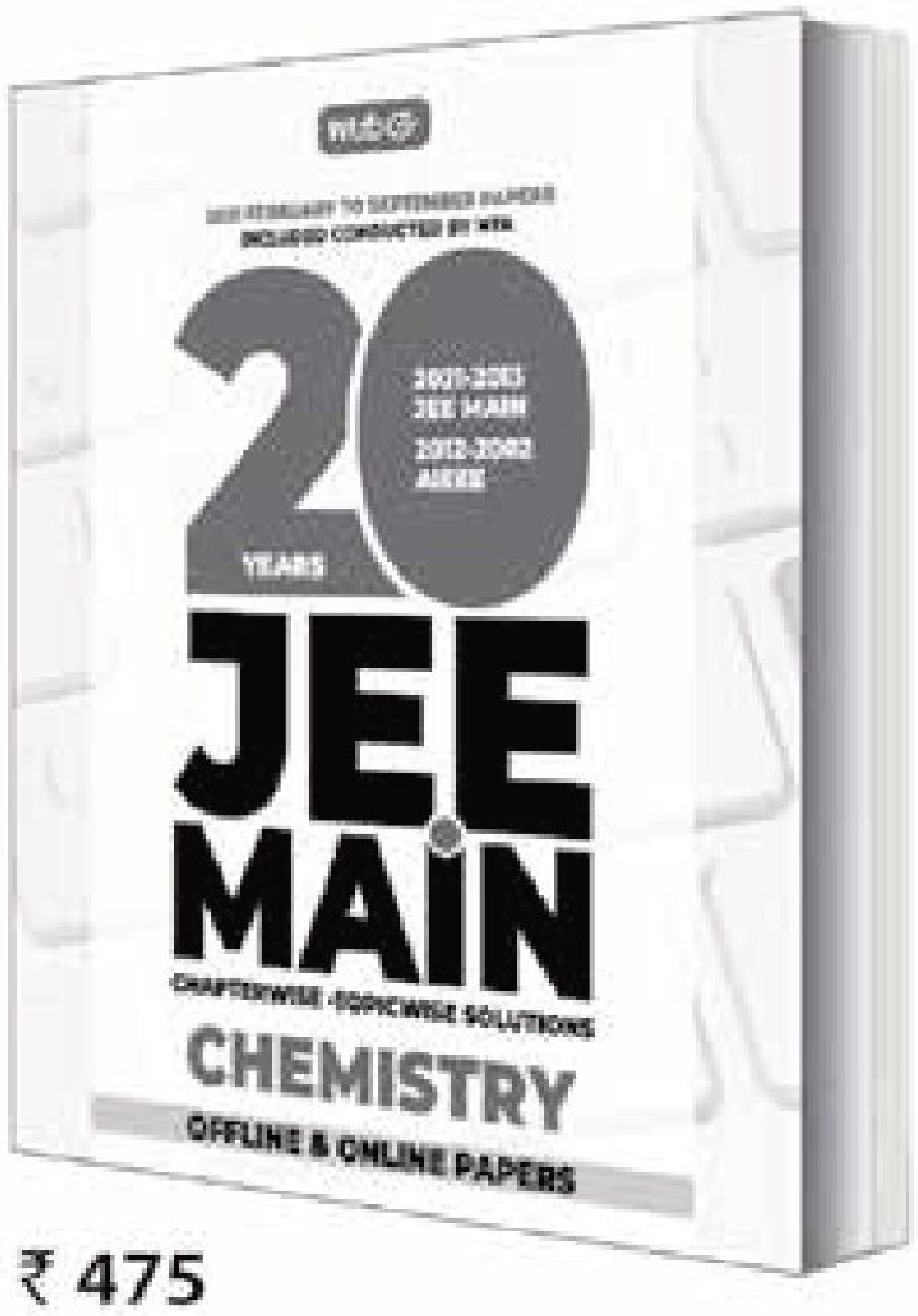
MtG

The Best tool for success in
JEE Main

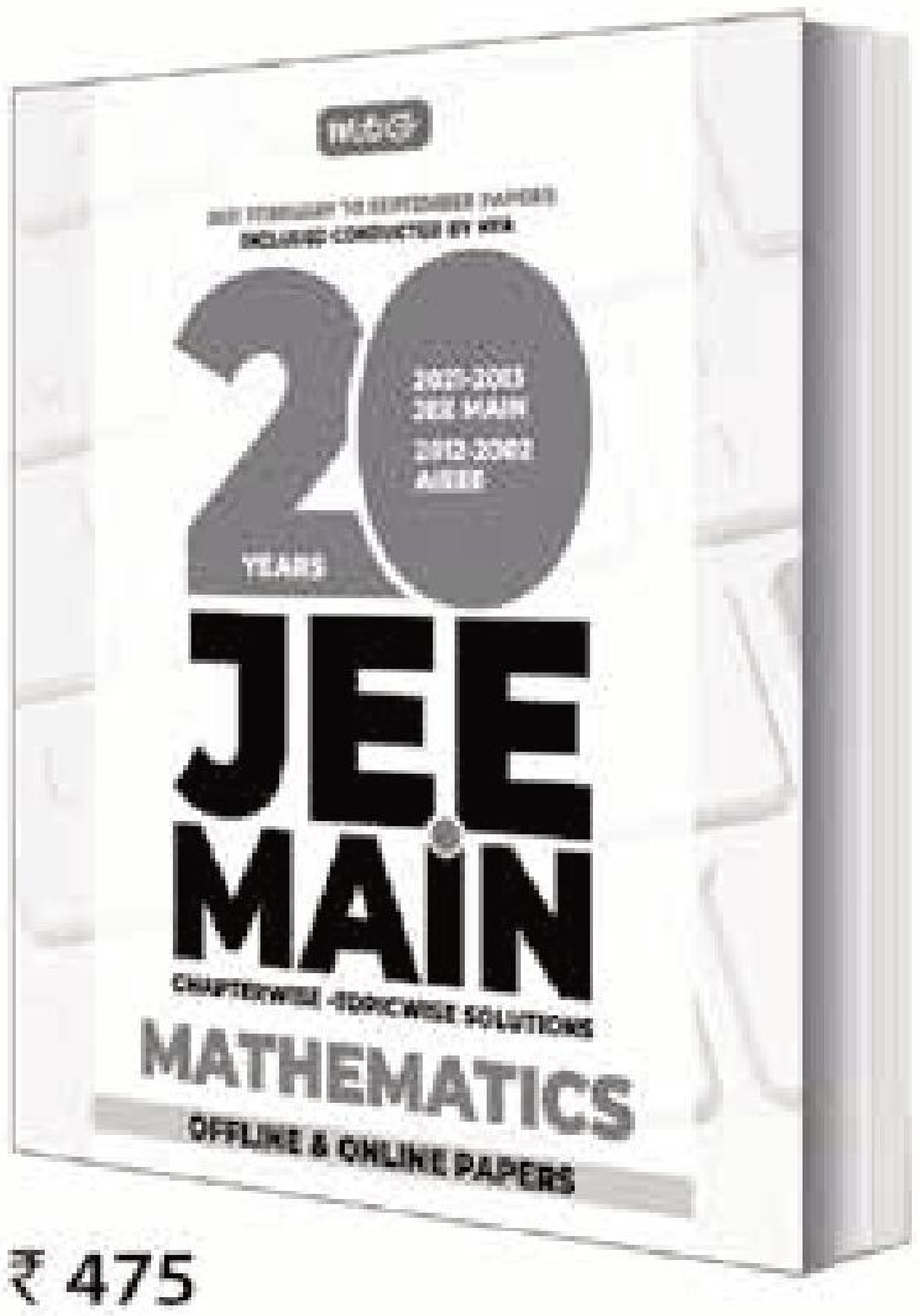
20 Years' JEE Main Chapterwise-Topicwise Solutions Physics, Chemistry and Mathematics contain not only chapterwise - topicwise questions that have appeared over the last 20 years in JEE Main / AIEEE but also their complete solutions. Needless to say these books are essential for any student to compete successfully in JEE Main.


MtG
20
YEARS
2021-2013 JEE MAIN
2012-2002 AIEEE
JEE MAIN
CHAPTERWISE-TOPICWISE SOLUTIONS
PHYSICS
OFFLINE & ONLINE PAPERS

₹ 475


MtG
20
YEARS
2021-2013 JEE MAIN
2012-2002 AIEEE
JEE MAIN
CHAPTERWISE-TOPICWISE SOLUTIONS
CHEMISTRY
OFFLINE & ONLINE PAPERS

₹ 475


MtG
20
YEARS
2021-2013 JEE MAIN
2012-2002 AIEEE
JEE MAIN
CHAPTERWISE-TOPICWISE SOLUTIONS
MATHEMATICS
OFFLINE & ONLINE PAPERS

₹ 475

HIGHLIGHTS:

- Chapterwise-Topicwise Questions (Online & Offline) of last 20 years' (2021-2002) of JEE Main / AIEEE
- Chapterwise-Topicwise segregation of questions to help you assess the level of effort required to succeed
- An unmatched series with more than 1552 pages filled with detailed solutions by experts

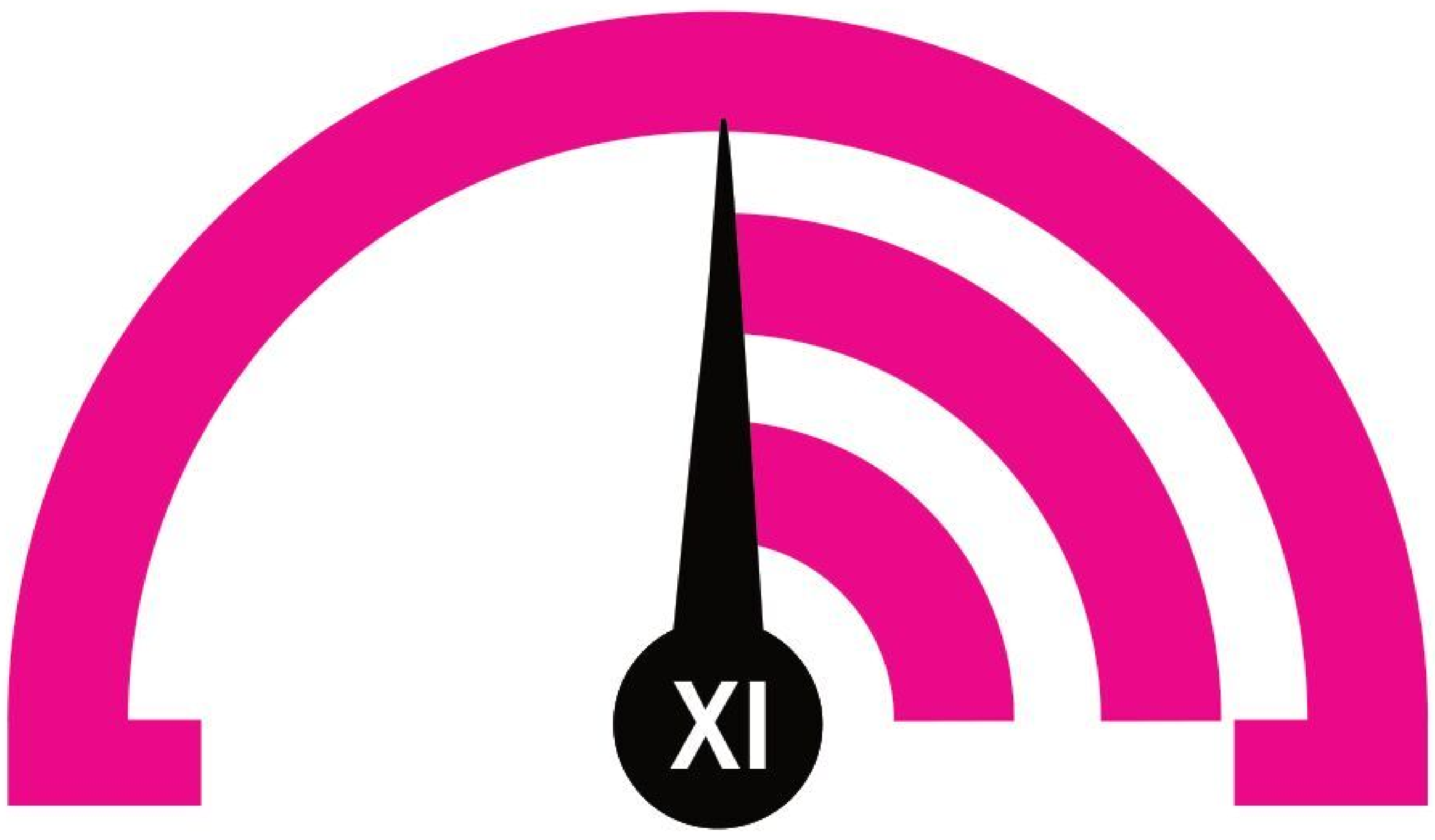
Visit
www.mtg.in
for latest offers
and to buy
online!

MTG Learning Media (P) Ltd.
Plot #99, Sector 44, Gurugram – 122 003 (HR)

Available at all leading book shops throughout India.
For more information or for help in placing your order,
Call 0124-6601200 or e-mail:info@mtg.in

MONTHLY TEST DRIVE

Practice Paper



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. If $f(x) = \dfrac{x-1}{x+1}$, then $f(2x)$ is equal to

(a) $\dfrac{3f(x)+1}{f(x)+3}$ (b) $\dfrac{f(x)+1}{f(x)+3}$

(c) $\dfrac{f(x)+3}{f(x)+1}$ (d) None of these

2. If $A = \cos^2\theta + \sin^4\theta$, then for all values of θ,

(a) $1 \le A \le 2$ (b) $\dfrac{13}{16} \le A \le 1$

(c) $\dfrac{3}{4} \le A \le \dfrac{13}{16}$ (d) $\dfrac{3}{4} \le A \le 1$

3. The equation $3(x + y - 5)^2 + 2(x - y + 7)^2 = 6$ represents an ellipse, whose centre is

(a) $(-1, 6)$ (b) $(6, -1]$

(c) $[1, -6)$ (d) $(-6, 1)$

4. $\lim\limits_{x\to 0}\left(\dfrac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ is equal to

(a) e (b) $e^{1/2}$

(c) e^{-2} (d) e^2

5. The domain of the function $y(x)$ given by the equation $2^x + 2^y = 2$ is

(a) $0 < x \le 1$ (b) $0 \le x \le 1$

(c) $-\infty < x \le 0$ (d) $-\infty < x < 1$

6. The number of solutions of

$\log 7 \log 5 \left(\sqrt{x+5}+\sqrt{x}\right) = 0$ is

(a) 0 (b) 1 (c) 2 (d) > 2

One or More Than One Option(s) Correct Type

7. If $\tan^3\theta + \cot^3\theta = 8\,\text{cosec}^3\,2\theta + 12$, then $\theta =$

(a) $n\pi - \dfrac{\pi}{12}$ (b) $n\pi + \dfrac{\pi}{12}$

(c) $n\pi - \dfrac{5\pi}{12}$ (d) $n\pi + \dfrac{5\pi}{12}$

8. If A, B are two events such that $P(A \cup B) \ge \dfrac{3}{4}$ and $\dfrac{1}{8} \le P(A \cap B) \le \dfrac{3}{8}$ then

(a) $P(A) + P(B) \le \dfrac{11}{8}$ (b) $P(A) \cdot P(B) \le \dfrac{3}{8}$

(c) $P(A) + P(B) \ge \dfrac{7}{8}$ (d) none of these

9. If a, b, c are in H.P., then the expression

$E = \left(\dfrac{1}{b}+\dfrac{1}{c}-\dfrac{1}{a}\right)\left(\dfrac{1}{c}+\dfrac{1}{a}-\dfrac{1}{b}\right)$ equals

(a) $\dfrac{2}{bc} - \dfrac{1}{b^2}$ (b) $\dfrac{1}{4}\left(\dfrac{3}{c^2}+\dfrac{2}{ca}-\dfrac{1}{a^2}\right)$

(c) $\dfrac{3}{b^2} - \dfrac{2}{ab}$ (d) none of these

10. Let $\tan x - \tan^2 x > 0$ and $|2 \sin x| < 1$. Then the intersection of which of the following two sets satisfies both the inequalities?

(a) $x > n\pi + \pi/6,\ n \in Z$ (b) $x > n\pi - \pi/6,\ n \in Z$

(c) $x < n\pi - \pi/4,\ n \in Z$ (d) $x < n\pi + \pi/4,\ n \in Z$

11. The integer part of the sum $\sum\limits_{n=1}^{16} \dfrac{n^4}{4n^2-1}$ is a multiple of

(a) 2 (b) 3

(c) 5 (d) 7

12. Let $y=\sqrt{x+\sqrt{x+\sqrt{x+....+\infty}}}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2y-1}$ (b) $\frac{x}{x+2y}$

(c) $\frac{1}{\sqrt{1+4x}}$ (d) $\frac{y}{2x+y}$

Comprehension Type

Paragraph for Q. No. 13 to 15

If n is positive integer and if $(1 + 4x + 4x^2)^n = \sum_{r=0}^{2n} a_r x^r$, where a_i is/are (i = 0, 1, 2, 3, ..., $2n$) real numbers.

13. The value of $2\sum_{r=0}^{n} a_{2r}$ is :

(a) $9^n - 1$ (b) $9^n + 1$

(c) $9^n - 2$ (d) $9^n + 2$

14. The value of $2\sum_{r=1}^{n} a_{2r-1}$ is :

(a) $9^n - 1$ (b) $9^n + 1$

(c) $9^n - 2$ (d) $9^n + 2$

15. The value of a_{2n-1} is :

(a) 2^{2n} (b) $(n - 1) \cdot 2^{2n}$

(c) $n \cdot 2^{2n}$ (d) $(n + 1) \cdot 2^{2n}$

Matrix Match Type

16. Match the following :

	Column-I		**Column-II**
P.	If maximum and minimum values of $\frac{7+6\tan\theta-\tan^2\theta}{(1+\tan^2\theta)}$ for all real values of $\theta \neq (2n+1)\frac{\pi}{2}$ are λ and μ respectively, then	1.	$\lambda + \mu = 2$
Q.	If maximum and minimum values of $5\cos\theta+3\cos\left(\theta+\frac{\pi}{3}\right)+3$ for all real values of θ are λ and μ respectively, then	2.	$\lambda - \mu = 6$
R.	If maximum and minimum values of $1+\sin\left(\frac{\pi}{4}+\theta\right)+2\cos\left(\frac{\pi}{4}-\theta\right)$ for all real values of θ are λ and μ respectively, then	3.	$\lambda + \mu = 6$
		4.	$\lambda - \mu = 10$
		5.	$\lambda - \mu = 14$

	P	**Q**	**R**
(a)	3, 4	3, 5	1, 2
(b)	3, 4	2, 3	1, 5
(c)	1, 2	4, 5	3
(d)	3, 4	1, 5	2

Numerical Answer Type

17. The number of integral values of a for which the point $(a - 1, a + 1)$ lies in the larger segment of the circles $x^2 + y^2 - x - y - 6 = 0$ made by chord whose equation is $x + y - 2 = 0$ is ________.

18. Let Z denotes the set of all integers where $A = \{(a, b): a^2 + 3b^2 = 28, a, b \in Z\}$ and $B = \{(a, b): a > b, a, b \in Z\}$, then the number of elements in $A \cap B$ is ________.

19. If the straight lines joining the origin to the points of intersection of $2x + 3y = k$ and $3x^2 - xy + 3y^2 + 2x - 3y - 4 = 0$ are at right angle, then $6k - \frac{52}{k}$ is ________.

20. The A.M. and S.D. of 100 items was recorded as 40 and 5.1 respectively. Later on it was discovered that one observation 40 was wrongly copied down as 50. Then the correct S.D is ________.

◆ ◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

Term-II Exam on 7th June

CLASS-XII

Practice Paper 2022

Time allowed : 2 Hours
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections - A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each.*
3. *Section - B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

SECTION - A

1. Find the integrating factor of the differential equation $(1-x^2)\dfrac{dy}{dx}-xy=1$.

2. If $\vec{a}=-3\hat{i}+n\hat{j}+4\hat{k}$ and $\vec{b}=-2\hat{i}+4\hat{j}+p\hat{k}$ are collinear, then find the values of n and p.

OR

If $\vec{a}=4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-2\hat{k}$, then find the value of $2\vec{b}\times\vec{a}$.

3. Find the value of $\displaystyle\int_{-\frac{\pi}{4}}^{0}\frac{1+\tan x}{1-\tan x}\,dx$

4. Evaluate : $\displaystyle\int(2\tan x-3\cot x)^2\,dx$

5. Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i}-3\hat{j}+6\hat{k}$.

6. If A and B are two independent events with $P(A)=\dfrac{1}{3}$ and $P(B)=\dfrac{1}{4}$, then find $P(B'|A)$.

SECTION - B

7. Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$.

8. Evaluate : $\displaystyle\int_{0}^{\pi/2}2\sin x\cos x\tan^{-1}(\sin x)\,dx$

OR

Evaluate : $\displaystyle\int\frac{\sin x-\cos x}{\sqrt{\sin 2x}}\,dx$

9. Find the particular solution of the differential equation $x\dfrac{dy}{dx}-y+x\sin\left(\dfrac{y}{x}\right)=0$, given that when $x=2$, $y=\pi$.

OR

Find the particular solution of the differential equation $\dfrac{dy}{dx}=\dfrac{xy}{x^2+y^2}$, given that $y=1$ when $x=0$.

10. Let $\vec{a},\vec{b},\vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\dfrac{\pi}{6}$. Prove that $\vec{a}=\pm 2(\vec{b}\times\vec{c})$.

SECTION - C

11. Using integration, find the area of the region in the first quadrant enclosed by the line $x + y = 2$, the parabola $y^2 = x$ and the x-axis.

12. Evaluate the integral : $\int \text{cosec}^3 x\,dx$

13. A coin is tossed three times, consider the following events :

A : 'No head appears'

B : 'Exactly one head appears'

C : 'At least two heads appear'

Do they form a set of mutually exclusive and exhaustive events?

OR

In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choice. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it, is 1/8. Find the probability that he knows the answer to the question, given that he correctly answered it.

CASE-BASED / DATA-BASED

14. The Indian Coast Guard (ICG) while patrolling, saw a suspicious boat with four men. They were nowhere looking like fishermen. The soldiers were closely observing the movement of the boat for an opportunity to seize the boat. They observe that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of coast guard helicopter and boat are (2, 3, 5) and (1, 4, 2) respectively.

Based on the above information, answer the following questions.

(i) If the line joining the positions of the helicopter and boat is perpendicular to the plane in which boat moves, then find the equation of plane.

(ii) If the soldier decides to shoot the boat at given instant of time, where the distance measured in metres, then what is the distance that bullet has to travel?

SOLUTIONS

1. We have, $(1 - x^2)\dfrac{dy}{dx} - xy = 1$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{x}{1-x^2}\cdot y = \frac{1}{1-x^2}$$

$$\therefore \quad \text{I.F.} = e^{-\int \frac{x}{1-x^2}dx} = e^{\frac{1}{2}\int \frac{-2x}{1-x^2}dx}$$

$$= e^{\frac{1}{2}\log(1-x^2)} = e^{\log(1-x^2)^{\frac{1}{2}}} = \sqrt{1-x^2}$$

2. We have, $\vec{a}$ and $\vec{b}$ are collinear

$$\therefore \ \vec{a} = \lambda\vec{b} \ \Rightarrow \ -3\hat{i} + n\hat{j} + 4\hat{k} = \lambda(-2\hat{i} + 4\hat{j} + p\hat{k})$$

$$\Rightarrow \lambda = \frac{3}{2}$$

Also, $n = 4\lambda \Rightarrow n = 4 \times \dfrac{3}{2} = 6$

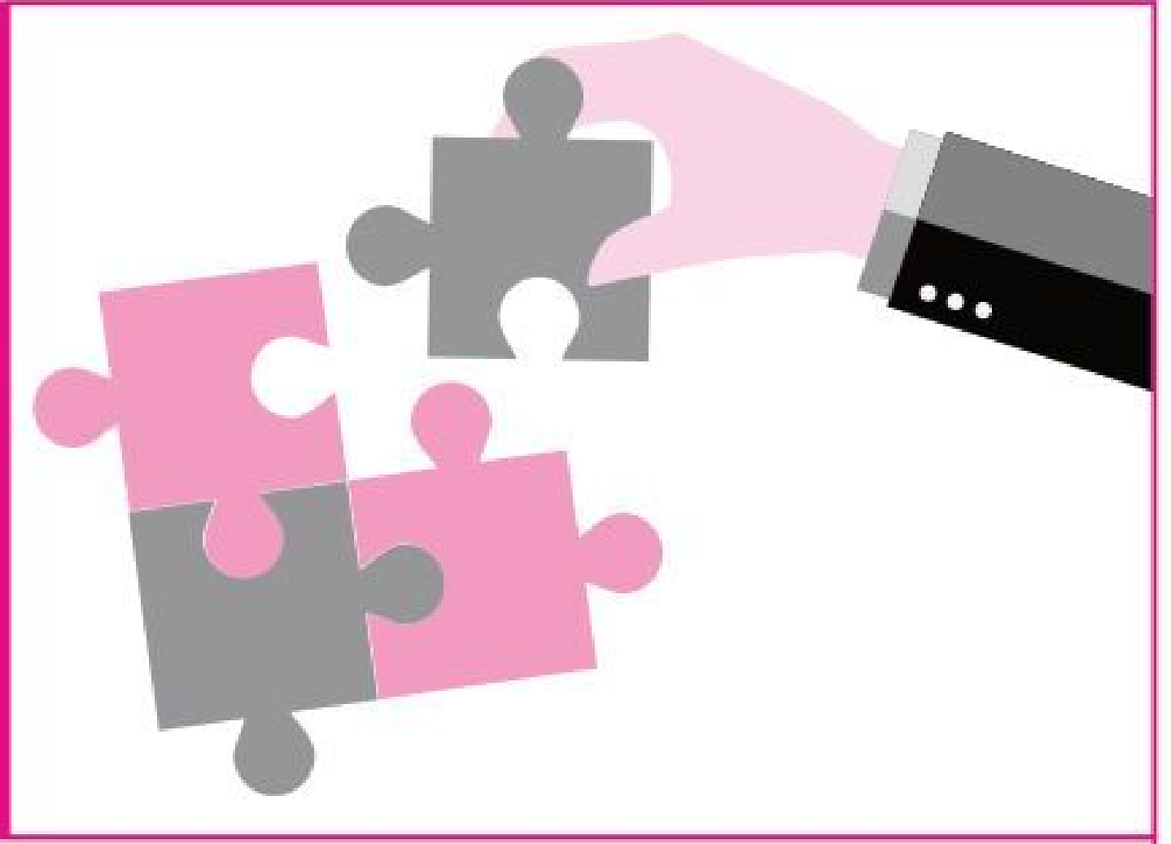
And, $\lambda p = 4 \Rightarrow p = 4 \times \dfrac{2}{3} = \dfrac{8}{3}$

OR

We have, $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{k}$

Now, $2\vec{b} = 2(\hat{i} - 2\hat{k}) = 2\hat{i} - 4\hat{k}$

PUZZLE CORNER

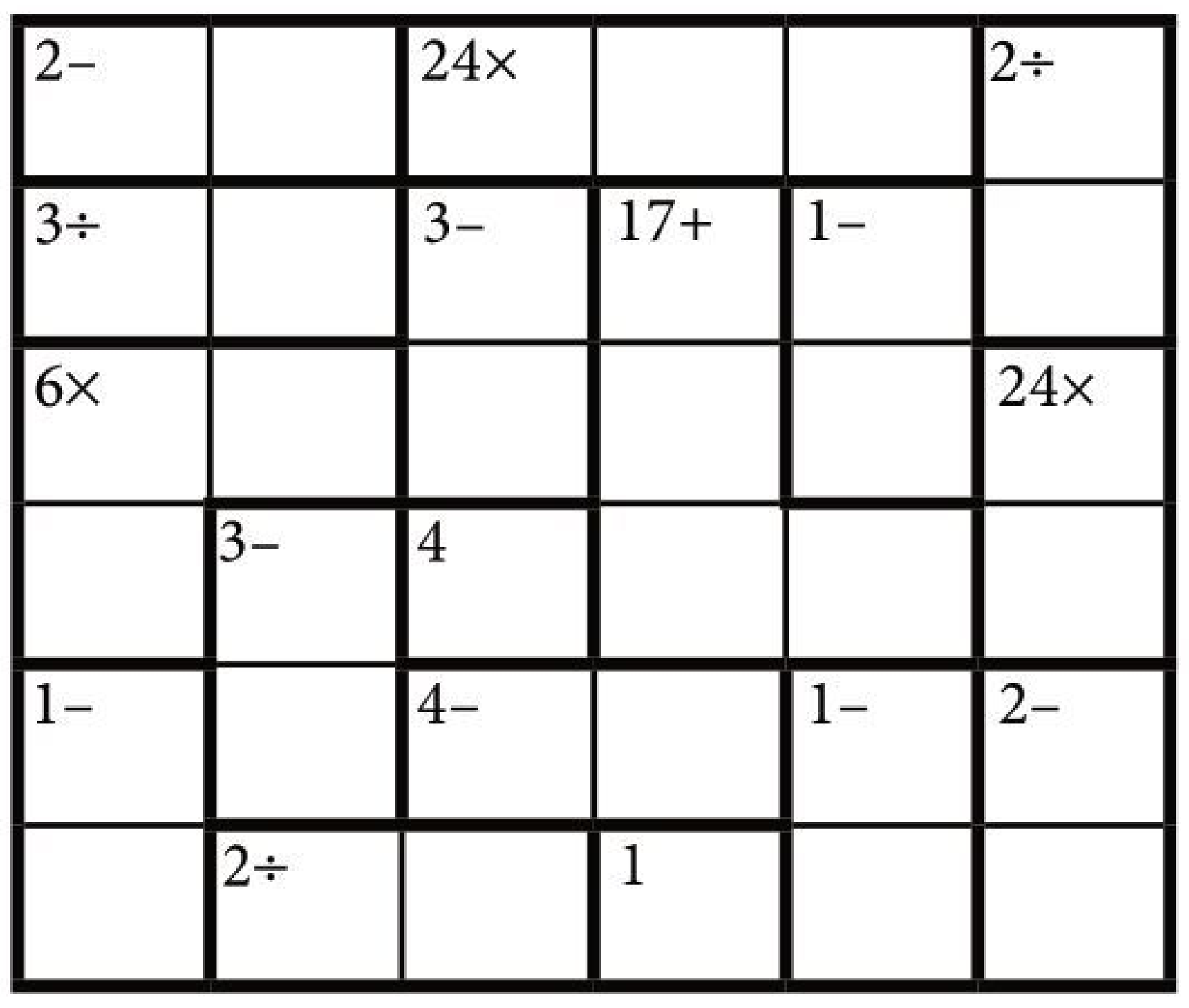


MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics.

In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is multiple of its numbers. For example, if that value is 3 for a two-box cluster, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



2–		24×			2÷
3÷		3–	17+	1–	
6×					24×
	3–	4			
1–		4–		1–	2–
	2÷		1		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$$\text{Now, } 2\vec{b}\times\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & -4 \\ 4 & 3 & 1 \end{vmatrix}$$

$$= (0+12)\hat{i} - (2+16)\hat{j} + (6-0)\hat{k} = 12\hat{i} - 18\hat{j} + 6\hat{k}$$

3. Let $I = \int_{\frac{-\pi}{4}}^{0} \frac{(1+\tan x)}{(1-\tan x)} dx = \int_{\frac{-\pi}{4}}^{0} \frac{\left(1+\frac{\sin x}{\cos x}\right)}{\left(1-\frac{\sin x}{\cos x}\right)} dx$

$$= \int_{\frac{-\pi}{4}}^{0} \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Put cos$x$ – sin$x$ = $t \Rightarrow$ –(sin$x$ + cos$x$) $dx = dt$

When $x = 0, t = 1$ and when $x = \frac{-\pi}{4}, t = \sqrt{2}$

$$\therefore \quad I = \int_{\sqrt{2}}^{1} -\frac{dt}{t} = \int_{1}^{\sqrt{2}} \frac{dt}{t} = [\log t]_1^{\sqrt{2}}$$

$$= \log\sqrt{2} - \log 1 = \frac{1}{2}\log 2$$

4. Let $I = \int (2\tan x - 3\cot x)^2 dx$

$\Rightarrow \quad I = \int (4\tan^2 x + 9\cot^2 x - 12\tan x \cot x) dx$

$\Rightarrow \quad I = \int \left\{4(\sec^2 x - 1) + 9(\operatorname{cosec}^2 x - 1) - 12\right\} dx$

$\Rightarrow \quad I = \int (4\sec^2 x + 9\operatorname{cosec}^2 x - 25) dx$

$\Rightarrow \quad I = 4\tan x - 9\cot x - 25x + C$

5. Let $\vec{n} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$$\therefore \quad \hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{\sqrt{4+9+36}} = \frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}$$

So, the required equation of the plane is

$$\vec{r}\cdot\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = 5 \quad \Rightarrow \quad \vec{r}\cdot(2\hat{i} - 3\hat{j} + 6\hat{k}) = 35$$

6. Given, A and B are independent events.

Also, $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$

Now, $P(B'|A) = \frac{P(B'\cap A)}{P(A)}$

$= \frac{P(B')P(A)}{P(A)}$ [$\because$ A, B are independent events]

$= P(B') = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$

7. Let P be the point with position vector $\hat{i} + 3\hat{j} + 4\hat{k} \equiv (1, 3, 4)$ and the plane is $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$

$\Rightarrow$ $2x - y + z + 3 = 0$ ...(i)

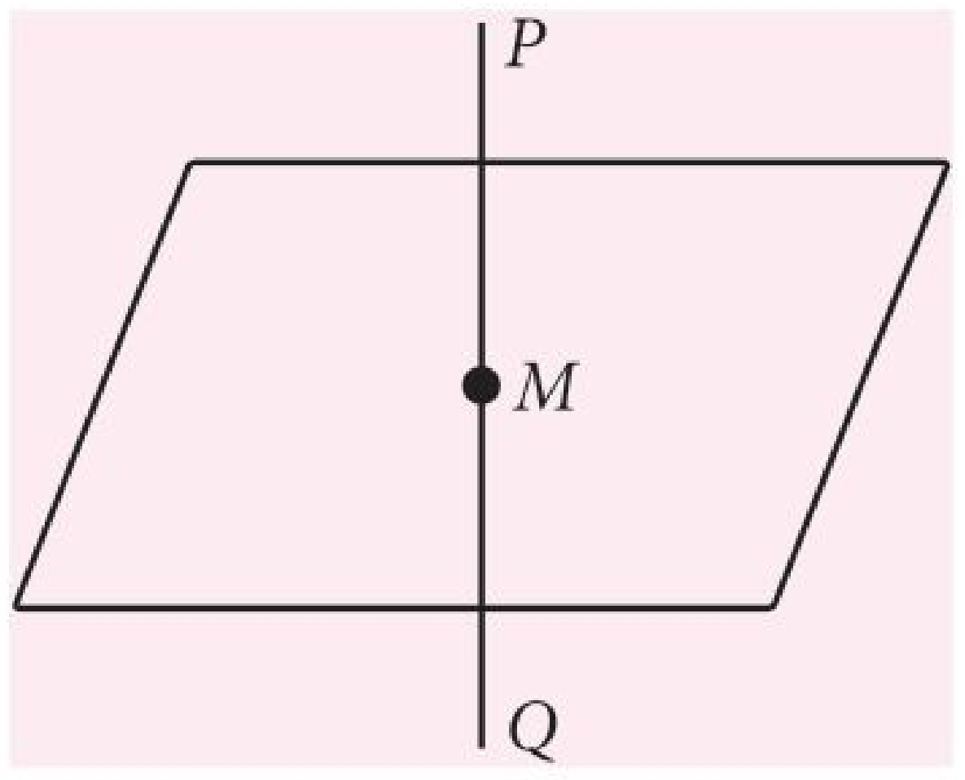
Any line $\perp$ to this plane passes through $P(1, 3, 4)$ is

$$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda \text{ (say)}$$

Any point M on the line is $(2\lambda + 1, -\lambda + 3, \lambda + 4)$.

This point lies on the plane (i).

$\therefore$ $2(2\lambda + 1) - (-\lambda + 3) + (\lambda + 4) + 3 = 0$



$\Rightarrow$ $6\lambda + 6 = 0 \Rightarrow \lambda = -1$

$\therefore$ Coordinates of point M are $(-1, 4, 3)$.

Let the image of P in the plane (i) be $Q(\alpha, \beta, \gamma)$, then M will be the midpoint of PQ.

$$\therefore \quad \frac{\alpha+1}{2} = -1, \frac{\beta+3}{2} = 4, \frac{\gamma+4}{2} = 3$$

$\Rightarrow$ $\alpha = -3, \beta = 5, \gamma = 2$

Hence, $Q(-3, 5, 2)$ is the image of $P(1, 3, 4)$ in plane (i).

8. Let $I = \int_{0}^{\pi/2} 2\sin x \cdot \cos x \tan^{-1}(\sin x) dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

Also, $x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = 1$

$$\therefore \quad I = 2\int_0^1 t \times \tan^{-1} t \cdot dt$$

$$= 2\left[\frac{t^2}{2} \times \tan^{-1} t\right]_0^1 - 2\int_0^1 \frac{1}{1+t^2} \times \frac{t^2}{2} \cdot dt$$

$$= 2 \times \frac{1}{2} \times \tan^{-1}\left(\tan\frac{\pi}{4}\right) - \int_0^1 \frac{1+t^2-1}{1+t^2} \cdot dt$$

$$= 1 \times \frac{\pi}{4} - \int_0^1 \left(1 - \frac{1}{1+t^2}\right) \cdot dt = \frac{\pi}{4} - [t - \tan^{-1} t]_0^1$$

$$= \frac{\pi}{4} - 1 + \tan^{-1}\left(\tan\frac{\pi}{4}\right) = \frac{2\pi}{4} - 1 = \left(\frac{\pi}{2} - 1\right)$$

OR

Let $I = \int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sqrt{1 + \sin 2x - 1}} dx$

$$= \int \frac{\sin x - \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x - 1}} dx$$

$$= \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} \cdot dx$$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)\, dx = dt$

$$\therefore \quad I=\int\frac{-dt}{\sqrt{t^2-1}}=-\log|t+\sqrt{t^2-1}|+C$$

$$=-\log|\sin x+\cos x+\sqrt{\sin 2x}\,|+C$$

9. We have, $x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0$

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x}+\sin\left(\frac{y}{x}\right)=0$$

This is a linear homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx}=v\cdot 1+x\frac{dv}{dx}$

$$\therefore \quad v+x\frac{dv}{dx}-v+\sin v=0$$

$$\Rightarrow x\frac{dv}{dx}+\sin v=0 \quad \Rightarrow \quad \operatorname{cosec} v\,dv+\frac{dx}{x}=0$$

Integrating both sides, we get

$\log|\operatorname{cosec} v - \cot v| + \log x = \log C$

$\Rightarrow \quad x(\operatorname{cosec} v - \cot v) = C$

$$\Rightarrow x\left[\operatorname{cosec}\left(\frac{y}{x}\right)-\cot\left(\frac{y}{x}\right)\right]=C$$

Now, when $x = 2, y = \pi$

$$\therefore \; 2\left[\operatorname{cosec}\frac{\pi}{2}-\cot\frac{\pi}{2}\right]=C\Rightarrow C=2$$

Therefore, $x\left[\operatorname{cosec}\left(\frac{y}{x}\right)-\cot\left(\frac{y}{x}\right)\right]=2$ is the required particular solution.

OR

We have, $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$ ...(i)

This is a homogeneous differential equation.

$$\therefore \quad \text{Put } y=vx \;\Rightarrow\; \frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\therefore \quad \text{From (i), } v+x\frac{dv}{dx}=\frac{x\cdot vx}{x^2+v^2x^2}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{v}{1+v^2} \;\Rightarrow\; x\frac{dv}{dx}=\frac{v}{1+v^2}-v$$

$$\Rightarrow x\frac{dv}{dx}=\frac{-v^3}{1+v^2} \quad\Rightarrow\quad \frac{dx}{x}=-\left(\frac{1+v^2}{v^3}\right)dv$$

Integrating both sides, we get

$$\log x = 1/2v^2-\log v+C$$

$$\Rightarrow \log x=\frac{x^2}{2y^2}-\log y+\log x+C$$

$$\Rightarrow \log y=\frac{x^2}{2y^2}+C$$

When $x = 0, y = 1 \;\Rightarrow\; \log 1 = 0 + C \Rightarrow\; C = 0$

$\therefore$ Particular solution is $\log y=\frac{x^2}{2y^2}\Rightarrow y=e^{\frac{x^2}{2y^2}}$

10. We have, $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\cdot\vec{c}=0$

$\Rightarrow \vec{a}\perp\vec{b}$ and $\vec{a}\perp\vec{c}$

$\Rightarrow$ $\vec{a}$ is perpendicular to the plane of $\vec{b}$ and $\vec{c}$.

$\Rightarrow$ $\vec{a}$ is parallel to $\vec{b}\times\vec{c}$

$\Rightarrow$ $\vec{a}=\lambda(\vec{b}\times\vec{c})$ for some scalar λ.

$$\Rightarrow |\vec{a}|=|\lambda||\vec{b}\times\vec{c}|\Rightarrow|\vec{a}|=|\lambda||\vec{b}||\vec{c}|\sin\frac{\pi}{6}$$

$$\Rightarrow 1=|\lambda|/2 \qquad \left[\because |\vec{a}|=|\vec{b}|=|\vec{c}|=1\right]$$

$$\Rightarrow |\lambda|=2 \Rightarrow \lambda=\pm 2$$

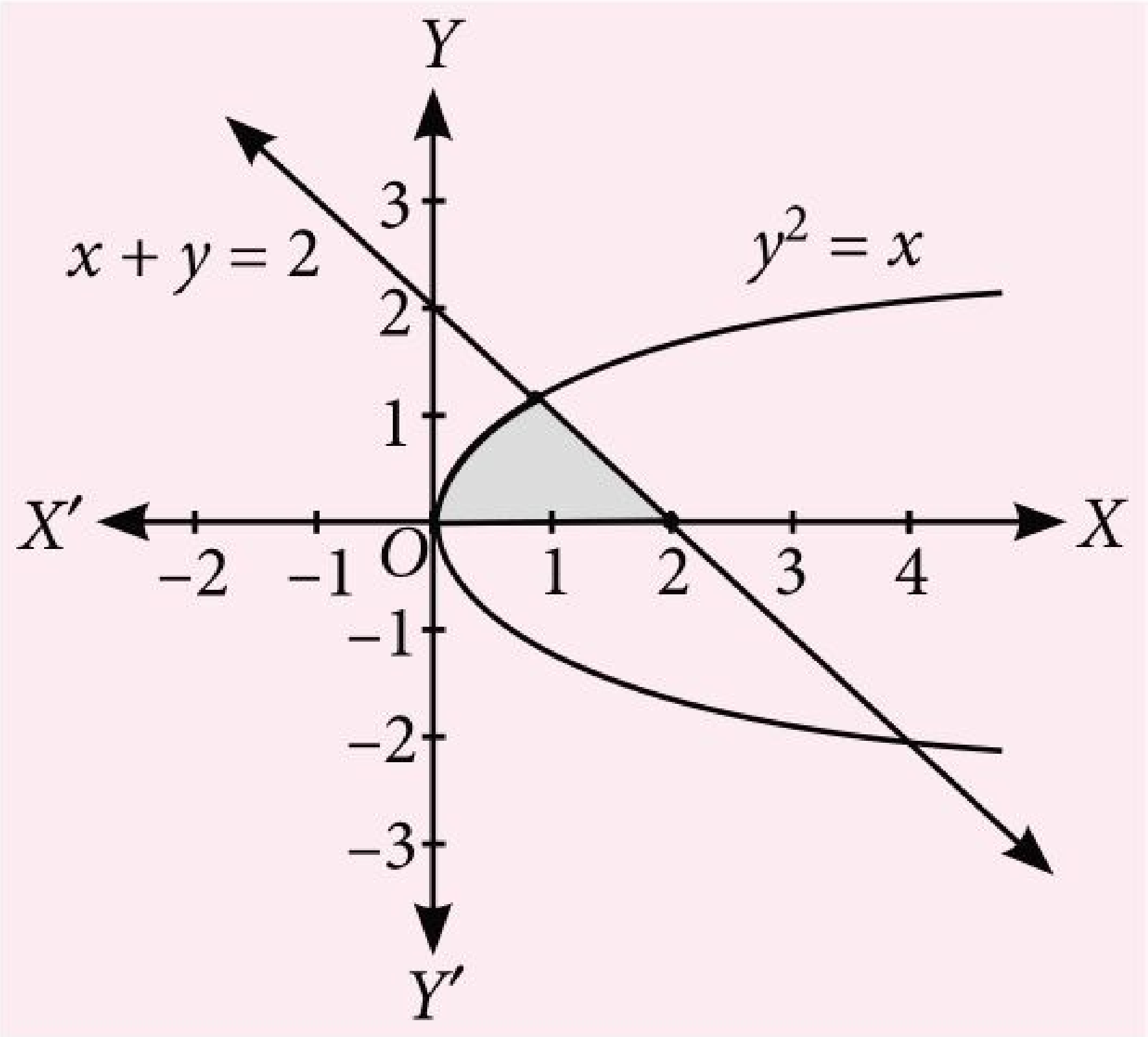
$$\therefore \vec{a}=\lambda(\vec{b}\times\vec{c}) \Rightarrow \vec{a}=\pm 2(\vec{b}\times\vec{c})$$

11. By solving $x + y = 2$ and $y^2 = x$ simultaneously, we get the points of intersection as (1, 1) and (4, –2).

SAMURAI SUDOKU

ANSWER · APRIL 2022

2	7	6	4	8	5	9	3	1				3	4	5	2	6	7	8	9	1
5	8	3	1	9	6	2	4	7				1	9	7	3	4	8	6	5	2
1	9	4	2	3	7	6	5	8				6	8	2	1	5	9	7	3	4
3	5	1	9	2	4	8	7	6				2	5	3	9	8	4	1	6	7
8	4	2	7	6	1	3	9	5				4	7	8	5	1	6	9	2	3
7	6	9	8	5	3	4	1	2				9	6	1	7	3	2	5	4	8
4	2	5	3	7	8	1	6	9	5	2	7	8	3	4	6	7	5	2	1	9
9	1	7	6	4	2	5	8	3	4	9	1	7	2	6	4	9	1	3	8	5
6	3	8	5	1	9	7	2	4	8	3	6	5	1	9	8	2	3	4	7	6
						2	4	5	6	8	9	3	7	1						
						3	1	6	7	4	2	9	5	8						
						8	9	7	3	1	5	4	6	2						
8	7	3	1	9	6	4	5	2	1	7	8	6	9	3	8	4	1	7	5	2
2	9	4	7	8	5	6	3	1	9	5	4	2	8	7	9	3	5	4	1	6
1	6	5	2	3	4	9	7	8	2	6	3	1	4	5	6	2	7	3	8	9
6	3	2	9	4	1	7	8	5				8	5	1	2	6	4	9	7	3
9	5	8	3	2	7	1	6	4				4	7	9	5	8	3	2	6	1
7	4	1	5	6	8	2	9	3				3	2	6	7	1	9	5	4	8
4	1	6	8	5	9	3	2	7				7	1	2	3	5	8	6	9	4
3	8	9	4	7	2	5	1	6				5	6	4	1	9	2	8	3	7
5	2	7	6	1	3	8	4	9				9	3	8	4	7	6	1	2	5



∴ Required area = The shaded area

$$= \int_0^1 \sqrt{x}\,dx + \int_1^2 (2-x)dx$$

$$= \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$= \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$ square units.

12. We have, $I = \int \operatorname{cosec}^3 x dx$

Then $I = \int \underset{\text{I}}{\operatorname{cosec} x}\, \underset{\text{II}}{\operatorname{cosec}^2 x}\, dx$

$$\Rightarrow I = \operatorname{cosec} x \int \operatorname{cosec}^2 x dx - \int \left(\frac{d(\operatorname{cosec} x)}{dx} \int \operatorname{cosec}^2 x dx \right) dx$$

$$\Rightarrow \quad I = -\operatorname{cosec} x \cot x - \int (\operatorname{cosec} x \cot x \cdot \cot x) dx$$

$$\Rightarrow \quad I = -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x dx$$

$$\Rightarrow \quad I = -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) dx$$

$$\Rightarrow \quad I = -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x dx + \int \operatorname{cosec} x dx$$

$$\Rightarrow \quad I = -\operatorname{cosec}x \cot x - I + \log|\operatorname{cosec}x - \cot x| + C$$

$$\Rightarrow \quad 2I = -\operatorname{cosec}x \cot x + \log|\operatorname{cosec}x - \cot x| + C$$

$$\therefore \quad I = \frac{1}{2}\left[\log|\operatorname{cosec} x - \cot x| - \operatorname{cosec} x \cot x\right] + C$$

13. If S be the sample space of tossing a coin three times, then S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Now, A (Event of getting no head) = {TTT}

B (Event of getting exactly one head)
= {HTT, THT, TTH}

C (Event of getting atleast two heads)
= {HHT, HHH, HTH, THH}

Clearly, $A \cap B = \phi$, $B \cap C = \phi$ and $A \cap C = \phi$

So, A, B and C are mutually exclusive events.

Also, $A \cup B \cup C = S$

So, A, B and C are exhaustive events.

OR

Let E_1, E_2, E_3 and A be the events defined as follows:
E_1 = the examinee guesses the answer, E_2 = the examinee copies the answer, E_3 = the examinee knows the answer and A = the examinee's answer correct.

$\therefore \quad P(E_1) + P(E_2) + P(E_3) = 1$

$\Rightarrow \quad P(E_3) = 1 - (P(E_1) + P(E_2)) = 1 - \frac{1}{3} - \frac{1}{6} = \frac{1}{2}.$

If E_1 has already occurred, then the examinee guesses. Since there are four choices out of which only one is correct, therefore the probability that he answered correctly given that he has made a guess is 1/4 *i.e.*, $P(A \mid E_1) = 1/4$. It is given that $P(A \mid E_2) = 1/8$, and $P(A \mid E_3)$ = probability that he answers correctly given that he knows the answer

$\Rightarrow \quad P(A \mid E_3) = 1$

By Bayes' theorem, we have

Required probability = $P(E_3 \mid A)$

$$= \frac{P(E_3)\, P(A|E_3)}{P(E_1)\, P(A|E_1) + P(E_2)\, P(A|E_2) + P(E_3)\, P(A|E_3)}$$

$$= \frac{\frac{1}{2} \times 1}{\frac{1}{3} \times \frac{1}{4} + \frac{1}{6} \times \frac{1}{8} + \frac{1}{2} \times 1} = \frac{24}{29}$$

14. **(i)** Let $P(2, 3, 5)$ and $Q(1, 4, 2)$ be the positions of helicopter and boat respectively.

Now, direction ratios of PQ are proportional to 1–2, 4–3, 2–5, *i.e.*, –1, 1, –3.

So, equation of plane passing through $Q(1, 4, 2)$ and perpendicular to PQ is

$-(x - 1) + (y - 4) + (-3)(z - 2) = 0 \Rightarrow x - y + 3z = 3$

(ii) Required distance = Distance between P and Q

$= \sqrt{(1-2)^2 + (4-3)^2 + (2-5)^2} = \sqrt{1+1+9} = \sqrt{11}$ m

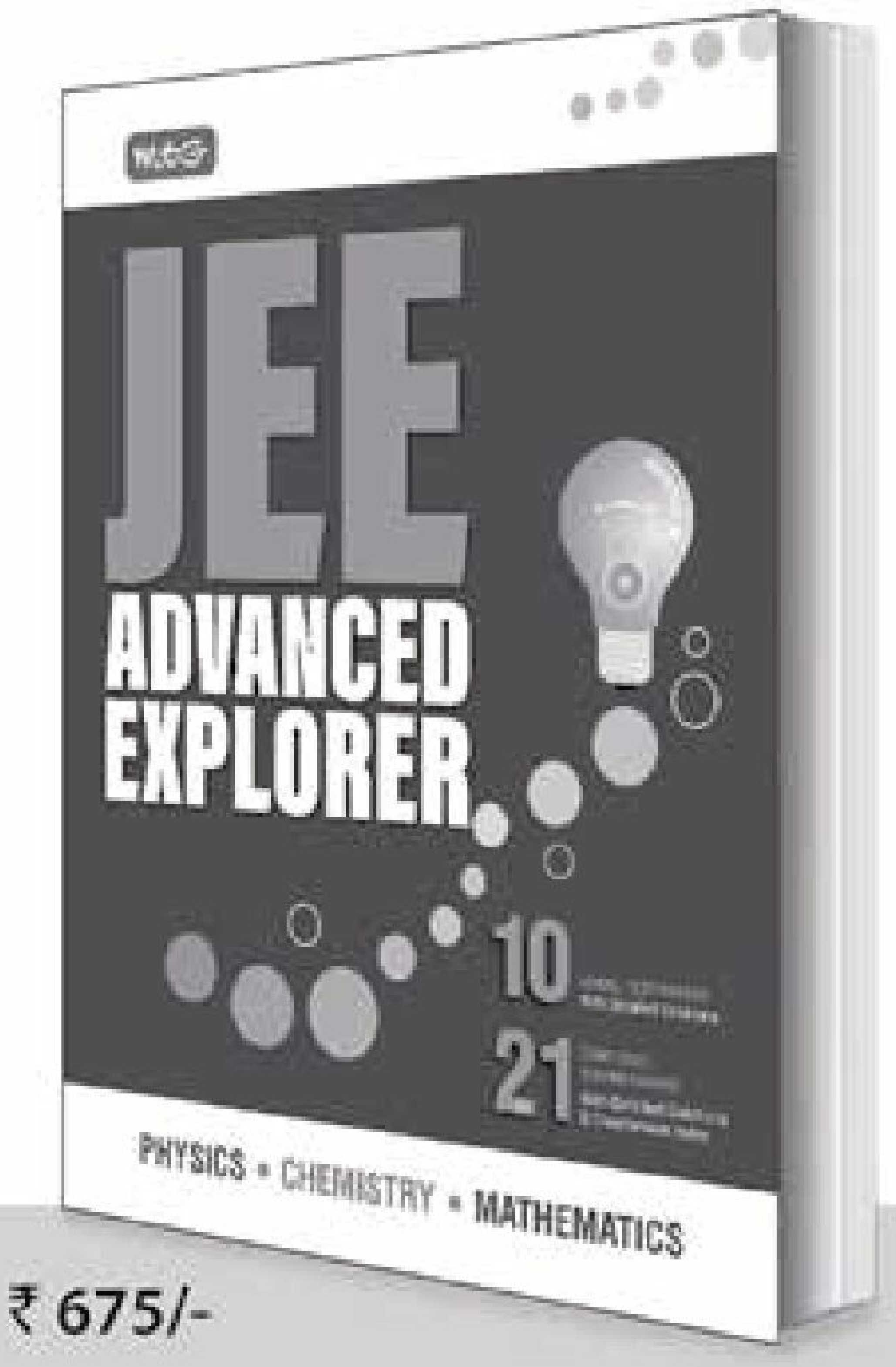
◆◆

EXAM ALERT 2022

Exam	Date
JEE Main	Session 1: 20th to 29th June Session 2: 21st to 30th July
BITSAT	Session-I: 20th to 26th June Session-II: 22nd to 26th July
COMEDK	19th June
SRMJEEE	Phase 3 : 25th and 26th June
KEAM	3rd July
VITEEE	30th June to 6th July
JEE Advanced	28th August

mtG

JEE
ADVANCED
EXPLORER


mtG
JEE
ADVANCED
EXPLORER
10
21
PHYSICS • CHEMISTRY • MATHEMATICS

21 years
(2001-2021)
Solved Papers
with detailed
Solutions &
chapterwise index

10 Model
Test Papers
with detailed
Solutions

₹ 675/-

Visit
www.mtg.in
for latest offers
and to buy
online!

mtG
Learning Media

Available at all leading book shops throughout the country.
For more information or for help in placing your order:
Call 0124-4951200 or email: info@mtg.in

MONTHLY TEST DRIVE

Practice Paper

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The value of $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right]$

(a) $-\frac{7}{17}$ (b) $-\frac{3}{17}$

(c) $-\frac{17}{3}$ (d) none of these

2. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b, c are the p^{th}, q^{th} and r^{th} terms of an H.P., is

(a) $ap + bq + cr$ (b) $(a + b + c)(p + q + r)$

(c) 0 (d) none of these

3. If $a < 0$, the function $f(x) = e^{ax} + e^{-ax}$ is a monotonically decreasing function for value of x given by

(a) $x > 0$ (b) $x < 0$

(c) $x > 1$ (d) $x < 1$

4. The value of $\int_0^{\infty}\frac{x\log x}{(1+x^2)^2}dx =$

(a) e (b) 1

(c) 0 (d) none of these

5. If $f(x)=\frac{\log(1+x)}{x}, x\neq 0$ is

(a) increasing for $x > 0$ (b) decreasing for $x > 0$

(c) increasing for all x (d) decreasing for all x

6. If $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10, 10)$ and

$f(x)=kf\left(\frac{200x}{100+x^2}\right)$, then $k =$

(a) 0.5 (b) 0.6

(c) 0.7 (d) 0.8

One or More than One Option(s) Correct Type

7. The value of $k\in R$ for which the system of equations $x + ky + 3z = 0$, $kx + 2y + 2z = 0$, $2x + 3y + 4z = 0$ admits a non-trivial solution, is

(a) 2 (b) 5/2

(c) 3 (d) 5/4

8. If $\int\frac{e^{x-1}\cdot 2x}{(x^2-5x+4)}dx = AF(x-1)+BF(x-4)+C$

and $F(x)=\int\frac{e^x}{x}dx$, then

(a) $A=\frac{-2}{3}$ (b) $B=\frac{4}{3}e^3$

(c) $A=\frac{2}{3}$ (d) $B=\frac{8}{3}e^3$

9. If $f(x)=\int\limits_0^x\frac{\sin t}{t}dt, x>0,$ then

(a) $f(x)$ has a local maxima at $x = n\pi$ $(n = 2k, k \in I)$

(b) $f(x)$ has a local minima at $x = n\pi$ $(n = 2k, k \in I)$

(c) $f(x)$ has neither maxima nor minima at $x = n\pi$ $(n \in I)$

(d) $f(x)$ has local maxima at $x = n\pi$ $(n = 2k - 1, k \in I)$

10. If $f(x)=\begin{cases}\dfrac{x}{1+e^{\frac{1}{x}}}, & x\neq 0\\ 0, & x=0\end{cases}$, then

(a) $f'(0^+) = 1$ (b) $f'(0^+) = 0$

(c) $f'(0^-) = 1$ (d) $f'(0^-) = 0$

11. Let X and Y be two events such that $P(X/Y)=\frac{1}{2}, P(Y/X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is(are) correct?

(a) $P(X\cup Y)=\frac{2}{3}$
(b) X and Y are independent
(c) X and Y are not independent
(d) $P(X^c\cap Y)=\frac{1}{3}$

12. The inverse of the function $y=\frac{10^x-10^{-x}}{10^x+10^{-x}}$ is

(a) $\log_{10}(2-x)$
(b) $\frac{1}{2}\log_{10}\frac{1+x}{1-x}$
(c) $\frac{1}{2}\log_{10}(2x-1)$
(d) $\log_{10}\left(\frac{1-x}{1+x}\right)^{-1/2}$

Comprehension Type

Paragraph for Q. No. 13 to 15

Let L_1 and L_2 be the lines $x+2y-z-3=0=3x-y+2z-1$ and $2x-2y+3z-2=0=x-y+z+1$, then

13. The angle between the lines is

(a) $\frac{\pi}{4}$
(b) $\cos^{-1}\frac{1}{3}$
(c) $\cos^{-1}\sqrt{\frac{2}{53}}$
(d) $\cos^{-1}\sqrt{\frac{2}{83}}$

14. The distance of the origin from the point of intersection of L_1 and L_2 is

(a) $\sqrt{22}$
(b) $\sqrt{33}$
(c) $2\sqrt{11}$
(d) $\sqrt{11}$

15. The distance of the origin from the plane through the lines is

(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{1}{2\sqrt{2}}$
(c) $\frac{1}{3\sqrt{2}}$
(d) $\frac{1}{4\sqrt{2}}$

Matrix Match Type

16. Let $f(x)=\begin{cases}\frac{5e^{1/x}+2}{3-e^{1/x}}, & x\neq 0\\ 0, & x=0\end{cases}$, then match the following.

Column-I		Column-II	
P.	$y=f(x)$ is	1.	continuous at $x=0$
Q.	$y=xf(x)$ is	2.	discontinuous at $x=0$
R.	$y=x^2f(x)$ is	3.	differentiable at $x=0$
S.	$y=x^{-1}f(x)$ is	4.	non- differentiable at $x=0$

	P	Q	R	S
(a)	1, 4	2, 4	1, 3	1, 4
(b)	2, 4	1, 4	1, 3	2, 4
(c)	2, 4	2, 4	1, 3	1, 3
(d)	2, 4	1, 4	1, 3	1, 4

Numerical Answer Type

17. The resultant of vectors $\vec{a}$ and $\vec{b}$ is $\vec{c}$. If $\vec{c}$ trisects the angle between $\vec{a}$ and $\vec{b}, |\vec{a}|=6, |\vec{b}|=4$ then $|\vec{c}|$ is ________.

18. If A is the area bounded by the curve $y=8x^2-x^5$ and x-axis, then the value of $3A/8$ is ________.

19. If $\int e^{\tan\theta}(1-\tan\theta)^2\cos^2\theta\, d\theta=Ae^{\tan\theta}\cos^B\theta+K$, then the value of $A+B$ is ________.

20. A team of 5 persons is to be selected out of 5 men, 3 women and 2 children. The probability that at least two of them are women is $\frac{k}{2}$, then $k=$ ________.

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

mtG

SUBSCRIBE NOW

SAVE ₹ 920/-*
with subscription plans

Get them uninterruptedly

SUBSCRIPTION FORM

☐ Student ☐ Teacher

Name: ____
____
Complete Postal Address: ____________________________
____
Pin Code: ______________ **Mobile No.:** ________________
Email: ____

Confirm your choice by placing ✓ tick-mark in relevant boxes.

SELECT PLAN:

9 Months ☐ 15 Months ☐ 27 Months ☐

Individual Magazines
- ☐ PHYSICS FOR YOU (P)
- ☐ CHEMISTRY TODAY (C)
- ☐ MATHEMATICS TODAY (M)
- ☐ BIOLOGY TODAY (B)

Get Best Result in Combinations
- ☐ PCB Combo
- ☐ PCM Combo
- ☐ PCMB Combo

Subscription Price:

Months	27 Months	15 Months	9 Months
Individual Magazines Physics | Chemistry | Mathematics | Biology	₹ 850/- (Save ₹ 230/-)	₹ 500/- (Save ₹ 100/-)	₹ 300/- (Save ₹ 60/-)
Combo of 3 Magazines PCM | PCB	₹ 2500/- (Save ₹ 740/-)	₹ 1400/- (Save ₹ 400/-)	₹ 900/- (Save ₹ 180/-)
Combo of 4 Magazines PCMB	₹ 3400/- (Save ₹ 920/-)	₹ 1900/- (Save ₹ 500/-)	₹ 1200/- (Save ₹ 240/-)
Courier Charges Add to your subscription amount for quicker & reliable delivery	₹ 600/-	₹ 450/-	₹ 240/-

Note: Magazines are dispatched by Book-Post on 4th of every month (each magazine separately)

Recommended by (optional)

Name of your teacher: ______________________________
Teacher's Mobile No.: ______________________________

*on cover price

Enclose Demand Draft favouring
MTG Learning Media (P) Ltd. payable at New Delhi.
Mail this Subscription Form to Subscription Department.

MTG Learning Media (P) Ltd., Plot 99, Sector 44, Gurugram-122 003 (HR)

Get Digital Editions of MTG Magazines on http://digital.mtg.in/

subscription@mtg.in
www.mtg.in
To subscribe online
0124-6601200

CELEBRATING 40 YEARS OF EXCELLENCE

Trust of Millions, Since 1982

www.mtg.in

@40/- each
80 pages in each

For NEET, JEE (Main & Advanced) and BOARDS

WHY MTG MAGAZINES?

Not every student can afford the hefty fees of coaching institutes yet dreams to clear NEET or JEE or CBSE exams with excellent marks. MTG makes it easy and affordable for all students to access world class content created by top faculties & experts across the globe through its Monthly Magazines and Books.

In these magazines you will get
- ✓ Quality Practice papers
- ✓ Concept Explanations
- ✓ Solved Papers
- ✓ Current Updates
- ✓ Fun Educational Activities

...and much more.

SCAN QR CODE & SUBSCRIBE MAGAZINE ONLINE

LIFETIME SUBSCRIPTION
Plan for teachers and special schemes and offer available for libraries and coaching institutes.
SMS MTG to 8800255334 to learn more.

mtG

Fine Tune your **CBSE Term-II** Preparation

with

100 PERCENT Exam Ready

CBSE BOARD 2021-22 TERM-II

CBSE Term-I Exam Ready had 99% Concept Coverage

HIGHLIGHTS OF THE SERIES

CHAPTERWISE

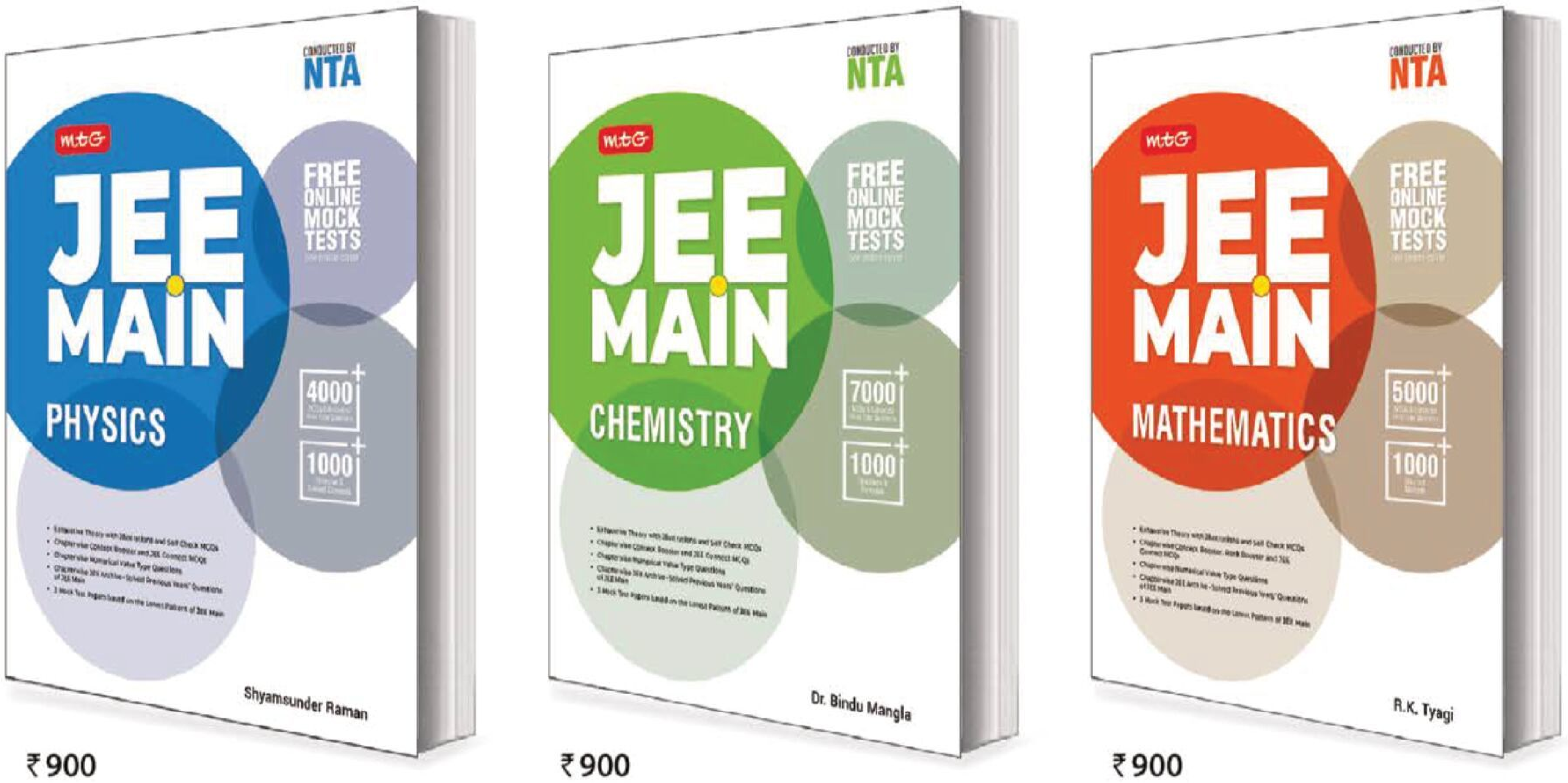
• RECAP NOTES • SUBJECTIVE QUESTIONS: SA I, SA II, LA & CASE BASED QUESTIONS
• 5 PRACTICE PAPERS WITH DETAILED SOLUTIONS
(CBSE SAMPLE QUESTION PAPER INCLUDED)

Available at all leading book shops throughout India. To buy online visit www.mtg.in
For more information or for help in placing your order, call 0124-6601200 or email info@mtg.in

Study right. Dig deep.

MtG

Build a solid foundation for success in JEE Main


CONDUCTED BY NTA
MtG
JEE MAIN
PHYSICS
CHEMISTRY
MATHEMATICS
FREE ONLINE MOCK TESTS
4000+
7000+
5000+
1000+
Shyamsunder Raman
Dr. Bindu Mangla
R.K. Tyagi

₹900 ₹900 ₹900

Are you a do-it-yourself type of a student? Then for success in JEE Main, choose MTG's JEE Main combo, comprising coursebooks for Physics, Chemistry & Mathematics. This combo is all class XI and XII students need for a solid and deep understanding of concepts in these three key subjects.

FEATURES

- Based on latest pattern of JEE Main
- Covers the entire syllabus
- Full of graphic illustrations for deep understanding of concepts
- Exhaustive Theory with Illustrations and Self Check MCQs
- Chapterwise Concept Booster, Rank Booster and JEE Connect MCQs
- Chapterwise Numerical Value Type Questions
- JEE Archive-Solved Previous Years' Questions of JEE Main
- 3 Mock Test Papers based on Latest Pattern of JEE Main

Scan now with your smartphone or tablet
Application to read QR codes required

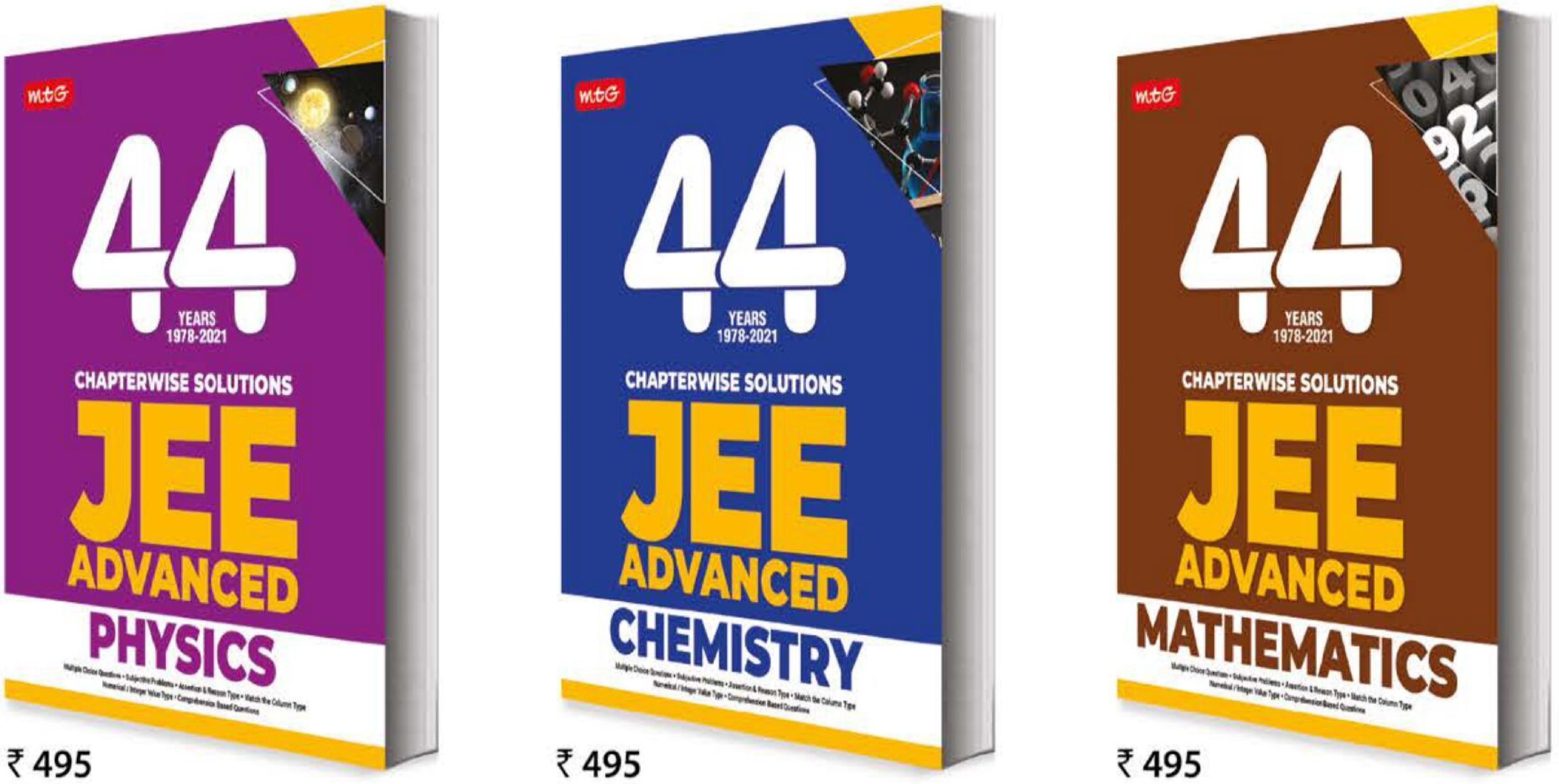
Note: Coursebooks are also available separately.

Available at all leading book shops throughout India. To buy online visit www.mtg.in.

For more information or for help in placing your order, call 0124-6601200 or e-mail: *info@mtg.in*

mtG

How can history help to succeed in JEE!



₹ 495 ₹ 495 ₹ 495

Wouldn't you agree that previous years' test papers provide great insights into the pattern and structure of future tests. Studies corroborate this, and have shown that successful JEE aspirants begin by familiarising themselves with problems that have appeared in past JEEs, as early as 2 years in advance.

Which is why the MTG team created 44 Years Chapterwise Solutions. The most comprehensive 'real' question bank out there, complete with detailed solutions by experts. An invaluable aid in your quest for success in JEE. Visit www.mtg.in to order online. Or simply scan the QR code to check for current offers.

Scan now with your smartphone or tablet

Application to read QR codes required

Note: 44 Years Chapterwise Solutions are also available for each subject separately.

Available at all leading book shops throughout India. To buy online visit www.mtg.in.

For more information or for help in placing your order, call 0124-6601200 or e-mail *info@mtg.in*

Registered R.N.I. No. 40700/1983 *Published on 1st of every month* Postal Regd. No. DL-SW-01/4045/21-23
Lic. No. U(SW)-29/2021-23 to post without prepayment of postage LPC Delhi RMS, Delhi-06 on 2-3rd same month

Your Gateway to **Success**

Aakash + BYJU'S

Medical (NEET)
Engineering (JEE)
Foundation (Class 8-10)

- 33+ years of delivering outstanding results
- 275+ branches & centres across India
- 100% Digitalized classrooms
- 85000+ Rankers from across India in NEET, JEE, NTSE & Olympiads

and more...

Appear for
Instant Admission
& Scholarship Test
Get up to 90% Scholarship*.

*Terms & Conditions Apply.

Hurry! Register today for FREE

To Register online visit:
iacst.aakash.ac.in
or scan the code

Download **Aakash App**
Google Play
App Store

CALL (TOLL-FREE)
1800-102-2727

VISIT
aakash.ac.in

Aakash + BYJU'S